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VEDIC MATHEMATICS

HIGHER APPLICATIONS

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आर्य

VEDIC MATHEMATICS

OR

SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

5975

Maths with Smiles

Higher Applications

Basic Concepts

Tribhujank

Calculas

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HIGHER APPLICATIONS OF VEDIC MATHEMATICS

by

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VM - HIGHER APPLICATION

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THE SIXTEEN SUTRAS OF VEDIC MATHEMATICS

सूत्र :-

१ एकाधिकेन पूर्वेण	1. Ekadhikena Purvena (also a corollary)
२ निखिलं नवतश्चरमं दशतः	2. Nikhilam Navatascaramam Dasatah
३ ऊर्ध्वतिर्यग्भ्याम्	3. Urdhva-tiryagbhyam
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५ शून्यं साम्यसमुच्चये	5. Sunyam Samyasamuccaye
६ (आनुरुप्ये) शून्यमन्यत्	6. (Anurupye) Sunyamanyat
७ संकलनव्यवकलनाभ्याम्	7. Sankalana-vyavakalanabhyam (also a corollary)
८ पूरणपूरणाभ्याम्	8. Puranapuranabhyam
९ चलनकलनाभ्याम्	9. Calana-Kalanabhyam
१० यावदूनम्	10. Yavadunam
११ व्यष्टिसष्टिः	11. Vyastisamastih
१२ शेषाण्यङ्केन चरमेण	12. Sesanyankena Caramena
१३ सोपान्त्यद्वयमन्त्यम्	13. Sopantyadvayamantyam
१४ एकन्यूनेन पूर्वेण	14. Ekanyunena Purvena
१५ गुणितसमुच्चयः	15. Gunitasamuccayah
१६ गुणकसमुच्चयः	16. Gunakasamuccayah

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प्रातः प्रभृति सायान्तम्
सायन्तः प्रातरन्तम्
यत् करोमि जगन्नाथ
तवस्तु तव पूजनम्

सुबह से शाम तक, शाम से सुबह तक,
मैं जो कुछ भी करता हूँ, हे जगन्नाथ
सब तेरी ही पूजा, तेरी ही उपासना हो जाए।

From morning till eve,
And evening till morn,
Whatever I do ,
Oh, Lord of Universe
Let that be my prayer of thee !

* * * * * * * * * * * * * * * *

ॐ तेजोऽसि तेजोमयि धेहि
वीर्यमसि वीर्यंमयि धेहि
ओजोऽसि ओजोमयि धेहि
बलमसि बलंमयि धेहि
अविराविमयी ।

हे तेज के पुन्ज हमें तेज दो।
हे वीर्य के पुन्ज हमें वीर्यवान बनाओ
हे ओजस्व स्वरूप हमें ओजस्व दो
हे बल के पुन्ज हमें बल प्रदान करो
हम अपने स्वरूप की ओर अग्रसर करें।

★ ★ ★ ★ ★ ★

ॐ सहनाववतु सहनौ भुनक्तु सह वीर्यं करवावहै।
तेजस्विनावधीतमस्तु मा विद्विषावहै।
ॐ शान्तिः शान्तिः शान्तिः।

हे परमात्मा हम गुरू और शिष्य दोनों की रक्षा करो। हम दोनों का पालन करो। हम दोनों साथ में रहकर तेजस्वी दैवी कार्य करें। हम दोनों का किया हुआ अध्ययन तेजस्वी दैवी हो और हम दोनों परस्पर एक दूसरे का द्वेष न करें।

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VEDIC MATHEMATICS SYSTEM

Whatever we see in nature has a perfect mathematical structure. The rotation of the earth, movement of the stars and planets, growth of the plants and trees etc. are all perfectly mathematically structured. The blooming of a flower, the growth of a plant, the rotation of the earth, all go on so very effortlessly. Recent research is bringing to light the perfect mathematical structure of the Vedic texts. As a matter of fact whatever we do in life has mathematics in some form or the other. It is not possible to prepare even a delicious cup of coffee without a precise sense and application of mathematical proportion of different ingredients used. It's a pity that such a wonderful subject of fundamental and vital importance 'mathematics' is creating the problem of 'MATHS ANXIETY' in all parts of the world. The common students are afraid of Mathematics. The gravity of the problem is quite clear from a recent survey report published in America. The top 10% of the students of Maths were asked the question, "Do you like Mathematics", And 60% of them said No. The answer of the remaining 90% is quite obvious.

Further, whatever we want to do, we always have a large number of alternatives and choices in doing the same. How strange that in Mathematics there seems to be almost no choice, and, most of the time we have to follow a unique, fixed, monotonous, often boring and tedious procedure in whatever arithmetical operation we want to do. Interestingly enough this unique approach is considered to be a quality by some mathematicians. Anything fixed, naturally becomes monotonous and boring and 'Maths Anxiety is the natural result. Vedic Maths on the other hand with its multiple choice makes maths a playful and interesting subject, which naturally brings frequent smiles on the face and joy in the heart of the students.

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The sixteen Vedic sutras provide a structured system which is applicable in, and, hence, is capable of explaining almost any facet of cosmos, both at abstract as well as applied level. These sutras in their philosophical interpretation even explain the functioning of human psychology. In their simple applied form, their crude translation in English has been presented in the beginning.

Sanskrit is a rich language and for deeper meaning one has to dive deep into the original sutras. The first Vedic Sutra Ekadhikena Purvena in simple English means by one more than the previous one, and a couple of very interesting and wonderful applications are presented in Pushp I. However the same sutra Ekadhikena Purvena (एकाधिकेन पूर्वेण) explains the whole concept of growth and development of an individual, society or any facet of knowledge. The word purvena essentially means the previous state. Ekadhiken means one higher than (the previous state). The higher state again becomes the previous and we go higher and higher and higher step by step, and that speaks out the whole process of growth and development. Similarly each sutra has a deep philosophical meaning. However, in the initial Pushps we are only looking at the simple applied facets of these sutras.

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(आऽस)

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VEDIC MATHEMATICS

OR

SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

Maths with Smiles

Tribhujank

वेदात्सर्वं प्रसिद्धयति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।

Vedas are the Eternal Source of Knowledge:

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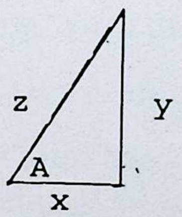
TRIBHUJANK

1.0 INTRODUCTION

The ancient Vedic rishis had complete knowledge and understanding of geometry also. This is very well accepted by all modern scholars. Here in we introduce ourselves to a very interesting, useful and thought provoking facet of ancient knowledge.A set of three numbers x, y and z which satisfy the condition $x^2 + y^2 = z^2$ is called Tribhujank (त्रिभुजांक). This word consists of three parts, Tri (Pronounced as Tree) means three; bhuja means the side; ank means the numbers. Therefore Tribhujank stands for a set of three numbers which essentially represents the three sides of a right angled triangle, having x as the base, y as the height and z as hypotenuse. Tribhujank written in this sequence essentially represents the angle (A) between the sides x and z.



The so called Pythagorus theorem was extensively used by the ancient Vedic people a few thousand years before the Pythagorus period.The following four formulae from Katyayan's sulbha sutra clearly demonstrate their perfect understanding of this simple and important facet of geometry.

1. सम चतुरस्रस्य अक्षण्या रज्जुः द्विकरणी ।
2. द्वि प्रमाणा चतुः करणी, त्रिः प्रमाणा नवकरणी, चतुः प्रमाणा षोडस करणी ।
3. अर्धप्रमाणेन पादप्रमाणम् विधीयते ।
4. तृतीयेन नवमोम्सः ।

meaning thereby

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1. The rope measuring the diagonal of a square can help in constructing the square twice in area as the first.

2. With the help of a rope twice the length of a side of a square, you can construct another of four times in area, thrice in length - nine times and four times in length- sixteen times.

3. Half the length of rope will yield quarter area.

4. One third length of rope will yield one ninth area.

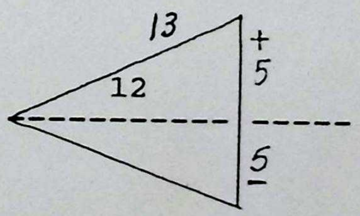
Even one old Babylonian tablet (Plimpton collection of Columbia University, New York) which dates from 1900- 1600 B.C. presents a set of fifteen Tribhujanks decreasing in steps of 1°.

The set of numbers 3, 4, 5 represent one Tribhujank as $3^2 + 4^2 = 5^2$. If we know the hypotenuse and any of the other two sides, the Vedic sutra (संकलन व्यकलनाभ्याम्) Sankalana Vyavakalanabhyam (No. 7) can be used to compute the third side very conveniently.The sutra means by addition and subtraction.

Example 1: If 13 is the hypotenuse (z) and 12 is the base (x) of a Tribhujank and the third side, height is y than

$$y^2 = z^2 - x^2$$

$$= (z+x)\ (z-x)$$



Therefore $y^2 = (13+12)\ (13-12)$

$$y^2 = 25$$

and $y = 5$.In general y can also be -5.(See Fig. 1.)

2.0 ADDITION AND SUBTRACTION OF TRIBHUJANK

Both of these operations can be easily done by direct application of the Urdhava sutra (No.3) which simply means

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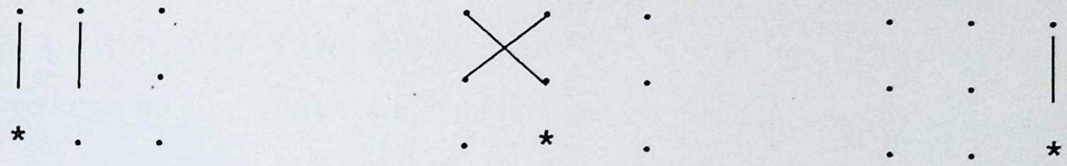
vertically and crosswise;only the specific procedural details have to be noted.The three elements of the new Tribhujank are obtained as follows:



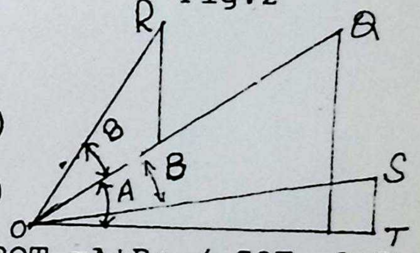
1st element   2nd element   3rd element

If (A) x1, y1, z1 and

(B) x2, y2, z2 are two Tribhujanks containing angles A & B respectively. Than the Tribhujank containing angles A - B and A + B are

A	x1	y1	z1
B	x2	y2	z2
A - B	(x1 x2+y1 y2)	(x2y1-x1y2)	(z1z2)
A + B	(x1x2-y1y2)	(x2y1+x1y2)	(z1z2)

Fig.2



∠ROT =A+B; ∠SOT =A-B

Let us consider one example.

Example 2: Find Tribhujank of angle A- B and A+ B. If

A	4	3	5
B	12	5	13
A - B	(4x12+3x5)	(12x3-4x5)	(5x13)
	63	16	65
Similarly;A+B	(4x12-3x5)	(12x3+4x5)	(5x13)
	33	56	65

Refer Fig. 2.

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NOTES:

1. Any common factor can be divided out.
2. These numbers essentially give us the desired angles.
3. If A is 4, 3, 5 then complement of angle A, (90- A) is simply represented by 3, 4, 5 as the third angle of the triangle is a right angle.

Example 3: Find the Tribhujank of 2A. If (A) is 4, 3, 5.

A	4	3	5
A	4	3	5
2A	(4x4-3x3)	(4x3+4x3)	(5x5)
OR	7	24	25
Similarly; A-A=0	16 + 9 25	12 - 12 0	25 25
0	1	0	1

Therefore Tribhujank of 0 is 1, 0, 1. (See Fig. 3).

The formulae for finding the Tribhujank for the double angle reduces to the simpler form. In general if

A	x	y	z
2A	$(x^2 - y^2)$	2xy	z^2

The first part can be directly obtained by using the seventh sutra meaning, 'by addition and subtraction.'

Example 4: Find the Tribhujank of A + B if A and B are complementary angles. Angle A is a irrational Tribhujank (A) 2, 3,

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$\sqrt{13}$, therefore B will be 3, 2, $\sqrt{13}$. We know the sum should be 90°.

A	2	3	$\sqrt{13}$
B	3	2	$\sqrt{13}$
A + B	6 - 6	9 + 4	13
	0	13	13
OR			
	0	1	1

Therefore Tribhujank of 90° is 0, 1, 1.

Similarly A - B is

12	5	13

Note that the sum and difference of two irrational Tribhujank need not essentially be irrational. (See Fig.4).

Similarly Tribhujank of 180° (which is double of 90°) is -1, 0, 1; of 270° is 0, -1, 1 and 360° is 1, 0, 1.These results have also been summarised in figures.(See Fig. 5).

Example 5: Find the Tribhujank for 2A.

A	5	12	13
2A	$5^2 - 12^2$	2x5x12	13^2
	- 119	120	169

The first element is negative as 2A is an obtuse angle.Therefore the base of the resultant triangle is extended in the opposite direction.(See Fig. 6).

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EX. 3

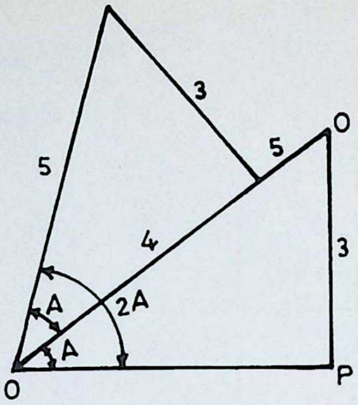

FIG. 3

EX. 4

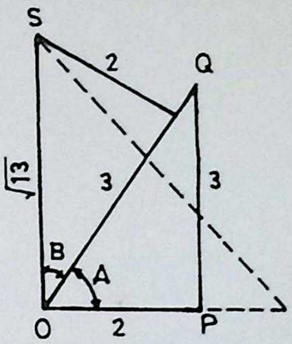

FIG. 4

EX. 5

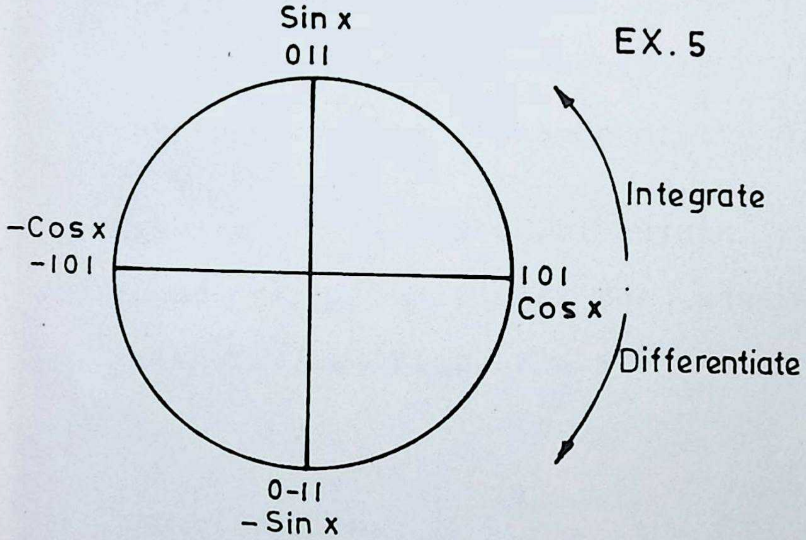

FIG. 5

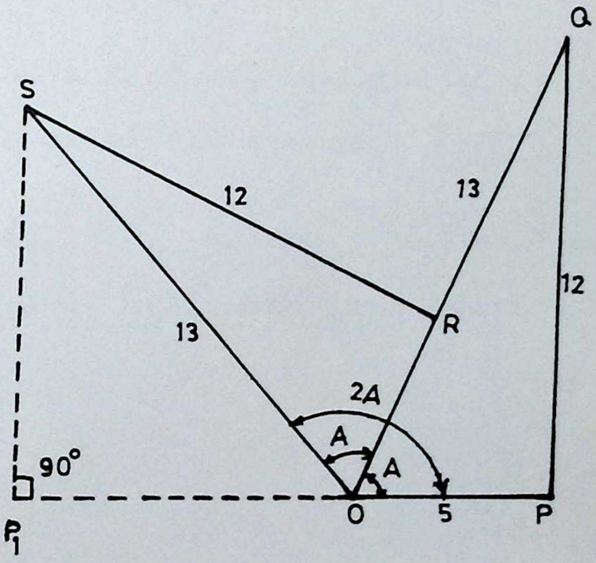

FIG. 6

EX. 6

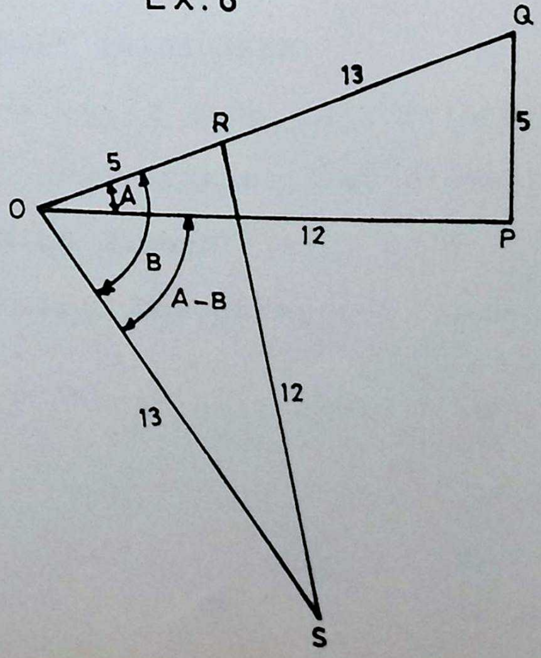

FIG. 7

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Example 6: Find the Tribhujank for A-B, where (A) 12, 5, 13 (b) 5, 12, 13

A	12	5	13
B	5	12	13
A-B	(60+60)	(25-144)	169
	120	- 119	169

The negative sign of the second element indicates that the resulting angle is negative.(See fig.7).

We can easily see that the Tribhujank of any angle (-A) can be obtained by simple change of the sign of middle element.

Example 7: The Tribhujank for 45°, 30°, 60° etc. can be easily found from half of the square and half of an equilateral triangle.(See Figs. 8 & 9).

45°	1	1	$\sqrt{2}$
30°	$\sqrt{3}$	1	2
60°	1	$\sqrt{3}$	2

3.0 **HALF ANGLE TRIBHUJANK**

If (A) x, y, z then A/2 Tribhujank will have the same middle element (y). However the first element shall be the addition of the first and third element (x+z) of A. Further,the third element can be computed by using the other two elements.

Example 8: Find the half angle Tribhujank of (A) 3, 4, 5.(Fig. 10).

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EX.7

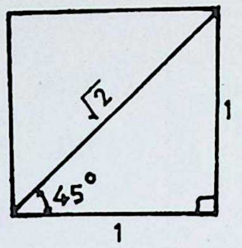

FIG.8

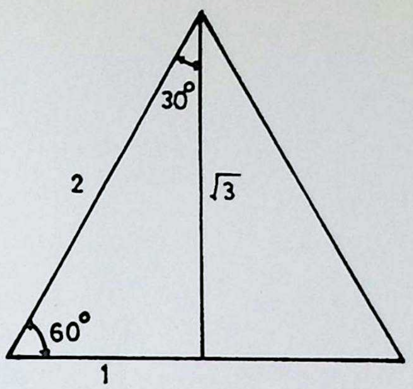

FIG.9

EX.8

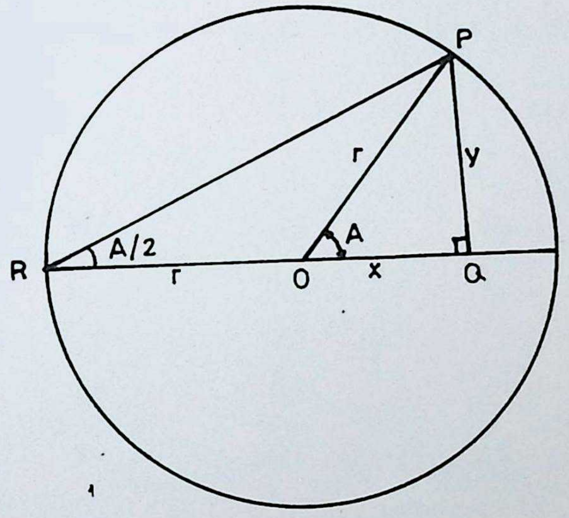

FIG. 10 & 11

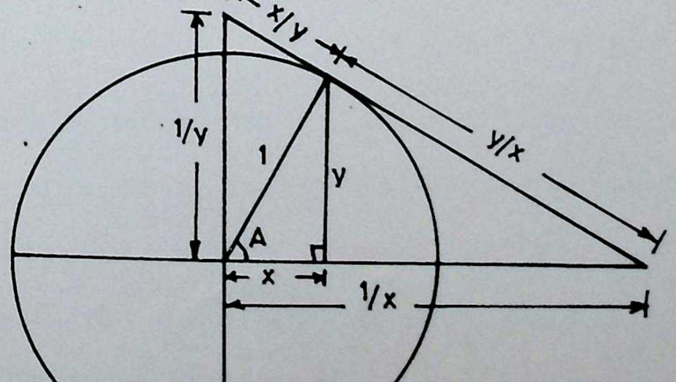

FIG.12

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A	3	4	5
A/2	3+5	4	-
OR	8	4	-
	2	1	$\sqrt{5}$

Both the geometric and algebraic proofs of this are given in Appendix a.

Example 9: Find the half angle Tribhujank for (A) 5, 12, 13. (Fig. 11).

A	5	12	13
A/2	5+13	12	-
	18	12	-
	3	2	$\sqrt{13}$

The numbers in the Tribhujank do not essentially give the actual dimensions of the new triangle and shall have to be suitably modified for applications in transformation and coordinate geometry.

4.0 TRIGONOMETRY

Trigonometry deals with the relations between sides and angles of triangles.We use six trigonometrical functions which are defined as six possible different ratios formed out of the three side of a right angled triangle. In general let any angle A be represented by Tribhujank x, y, z.Then we know that

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Function	Abbreviated as	Formulae
sine A	sin A	y/z
cosine A	cos A	x/z
tangent A	tan A	y/x
cosecant A	cosec A	z/y
secant A	sec A	z/x
cotangent A	cot A	x/y

The usual methods of proving identities, solving equations, computing ratios etc. in trigonometry, we have to use the appropriate formulae from a long list. But if we use the general form of Tribhujank (reducing z=1) we do not need any of the formulae. Further,even the use of the trigonometrical functions is not necessary. If A is any angle represented by the general Tribhujank p, q, i, and A can take any value between 0° and 360°. The six trigonometrical functions then can be defined as cosA = p; sinA = q; tanA = q/p ; cotA = p/q; secA = 1/p and cosecA = 1/q. We can also represent them on a single diagram.(Refer Fig. 12).

Example 10: If sinA = 4/5 and sin B = 12/13 find (i) tan(A+B); (ii) sin(A-B); (iii) cos(A+B); (iv) cos(2A); (v) cot(1/2A); (vi) sin(90-A) and (vii) tan(-A).

The third element of each Tribhujank is obtained by using the seventh Vedic sutra, which means by addition and subtraction.

(A) - , 4, 5

Base element $= \sqrt{(4+5)\ (5-4)}$

$= \sqrt{9}$

$= 3$ Therefore (A) is 3, 4, 5.

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Similarly (B) is 5, 12, 13.

	A	3	4	5	
	B	5	12	13	
(i)	A + B	(3x5-4x12) -33	(5x4+12x3) 56	13x5 65;	$\tan(A+B)=\frac{-56}{33}$
(ii)	A - B	-	-16	65;	$\sin(A-B)=\frac{-16}{65}$
	It is not necessary to compute all the three elements				
(iii)	A + B			;	$\cos(A+B)=\frac{-33}{65}$
(iv)	2A	$3^2 - 4^2$	-	5^2	
		-7	-	25;	$\cos 2A=\frac{-7}{25}$
(v)	A/2	3 + 5	4	-	
		8	4	-	
		2	1	-	(A/2) = 2
(vi)	90°	0	1	1	
	A	3	4	5	
	90° -A	-	3	5;	$\sin(90^{o}-A)=\frac{3}{5}$
(vii)	0°	1	0	1	
	A	3	4	5	
	0° -A	3	-4	-;	$\tan(-A)=\frac{-4}{3}$

4.1 **Inverse Functions:**

It is interesting to observe that handling of inverse functions

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involves absolutely nothing new.

Example 11: Evaluate (i) $\cot^{-1}(2)+\cot^{-1}(3)$,

(ii) $2\tan^{-1}(3)+\cot^{-1}(2)$

(iii) $\sin^{-1}(3/5)+\tan^{-1}(2)$

We are required to add the angles represented by the individual inverse functions.

(i)

A	2	1	5
B	3	1	$\sqrt{10}$
A + B	5	5	-
	1	1	$\sqrt{2}$

Therefore A + B = 45°

(ii)

A	1	3	$\sqrt{10}$
2A	-8	6	10
B	2	1	$\sqrt{5}$
2A + B	-22	4	-
	-11	2	-

Therefore $2\tan^{-1}(3)+\cot^{-1}(2) = \tan^{-1}(-2/11)$

(iii)

A	4	3	5
B	1	2	$\sqrt{5}$
A + B	-2	11	$5\sqrt{5}$

Therefore $\sin^{-1}(3/5)+\tan^{-1}(2) = \tan^{-1}(-11/2)$

4.2 Further Illustrations

Example 12: Evaluate Tribhujank for 120°, 240°, 675°, 105°, -15°,

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15°.

Tribhujank for 30° is $\sqrt{3}$, 1, 2

90° is 0, 1, 1

Therefore Tribhujank for 120° is -1 $\sqrt{3}$ 2

Tribhujank for

60°	1	$\sqrt{3}$	2
180°	-1	0	1
240°	-1	$-\sqrt{3}$	2

Tribhujank for

360°	1	0	1
720°	1	0	1
45°	1	1	$\sqrt{2}$
$720-45=675^\circ$	1	-1	$\sqrt{2}$

Tribhujank for

45°	1	1	$\sqrt{2}$
60°	1	$\sqrt{3}$	2
$45^\circ + 60^\circ = 105^\circ$	$1-\sqrt{3}$	$1+\sqrt{3}$	$2\sqrt{2}$
$45-60=$ (-15)	$1+\sqrt{3}$	$1-\sqrt{3}$	$2\sqrt{2}$

Tribhujank for 15° can be obtained by two alternative ways.

60°	1	$\sqrt{3}$	2
45°	1	1	$\sqrt{2}$
60-45=15	$1+\sqrt{3}$	$\sqrt{3}-1$	$2\sqrt{2}$

OR

30°	$\sqrt{3}$	1	2
$30^\circ/2 = 15^\circ$	$2+\sqrt{3}$	1	–

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Example 13: Prove the following :

(i) $$\frac{\sin A + \sin 2A}{\cos A - \cos 2A} = \frac{\cot A/2}{2}$$

(ii) $$\frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan(A-B)$$

(iii) $$\sin 105^o + \cos 105^o = \cos 45^o$$

Let Tribhujank for (A) be x y 1

A	x	y	1
2A	$x^2 - y^2$	2xy	1
A/2	x+1	y	$\sqrt{(x+1)^2 + y^2}$

Similarly if (B) x1, y1. 1

A	x	y	1
B	x1	y1	1
A - B	(x.x1+y.y1)	(x1.y-x.y1)	1
2B	$(x1^2 - y1^2)$	2x1y1	1

(i) $$\frac{\sin A + \sin 2A}{\cos A - \cos 2A} = \frac{y + 2xy}{x - (x^2 - y^2)}$$

$$= \frac{y(1 + 2x)}{x + 1 - 2x^2} \quad \left[\text{as } y^2 = 1 - x^2 \right]$$

$$= \frac{y}{1 - x} * \frac{1 + x}{1 + x}$$

$$= \frac{x + 1}{y} = \cot(A/2)$$

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(ii) $$\frac{\cos 2B - \cos 2A}{\sin 2B + \sin 2A} = \tan(A-B)$$

$$\frac{x1^2 - y1^2 - x^2 + y^2}{2x1.y1 + 2xy} = \frac{x1.y - x.y1}{x.x1 + y.y1}$$

$$\frac{x1^2 - x^2}{x1.y1 + xy} = \frac{y.x1 - x.y1}{x.x1 + y.y1} \quad \left[\text{as } y^2 = 1 - x^2 \right]$$

By crossmultiplying we get,

$$x1^2 + y1^2 = x^2 + y^2 = 1 \quad \text{(Okay)}$$

(iii) $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$

The Tribhujank of 105° is $1-\sqrt{3}$, $1+\sqrt{3}$, $2\sqrt{2}$ (calculated in the previous question 12).

Therefore

$$\frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

Proved.

Present Method :

$$\sin 105^\circ + \cos 105^\circ = \sin(180^\circ - 105^\circ) - \cos(180^\circ - 105^\circ)$$
$$= \sin 75^\circ - \cos 75^\circ$$
$$= \sin 75^\circ - \sin(90^\circ - 75^\circ)$$
$$= \sin 75^\circ - \sin 15^\circ$$
$$= 2\cos 45^\circ \sin 30^\circ$$
$$= 2 \cos 45^\circ * \frac{1}{2}$$

* (a series of formulae have been used)

$$= \cos 45^\circ$$

proved.

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Example 14: Prove the following :

(i) $$\frac{\cos A+\sin A}{\cos A-\sin A} - \frac{\cos A-\sin A}{\cos A+\sin A} = 2\tan 2A$$

(ii) $$\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$$

(iii) $$\sin(45^\circ+A)\,\sin(45^\circ-A) = (1/2)\cos 2A$$

(iv) $$\sec A + \tan A = \tan(45^\circ + A/2)$$

(v) $$\sin A\,\sin(60-A)\,\sin(60+A) = (1/4)\sin 3A$$

Solutions :

The Tribhujank for A

A	x	y	1
2A	x^2-y^2	$2xy$	1

(i) For L.H.S. $$\frac{x+y}{x-y} - \frac{x-y}{x+y}$$

$$= \frac{4xy}{x^2-y^2} = 2\tan 2A.$$ R.H.S. Proved.

(ii) For L.H.S.

$$\frac{1-(x^2-y^2)}{1+(x^2-y^2)} = \frac{y^2}{x^2} = \tan^2 A$$ R.H.S. Proved.

(iii) The Tribhujank for

45°	1	1	$\sqrt{2}$
A	x	y	1
$45^\circ+A$	$x-y$	$x+y$	$\sqrt{2}$
$45^\circ-A$	$x+y$	$x-y$	$\sqrt{2}$

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For L.H.S.

$$\sin(45 + A)\ \sin(45 - A) = \left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{x-y}{\sqrt{2}}\right)$$

$$= \frac{x^2 - y^2}{2} = (\cos 2A)/2 \quad \text{R.H.S.}$$

(iv) The Tribhujank for

A	x	y	1
A/2	x+1	y	-
45°	1	1	$\sqrt{2}$
45+A/2	(x+1-y)	(y+x+1)	-

For L.H.S. $\sec A + \tan A = \frac{1}{x} + \frac{y}{x}$

$$= \frac{1+y}{x} = \text{R.H.S.}$$

R.H.S. = (y + x + 1)/ (x − y + 1)

Cross multiply and transpose

$$x^2 + y^2 = 1$$

$$1 = 1 \qquad \text{Proved.}$$

(v) The Tribhujank of

60°	1	$\sqrt{3}$	2
A	x	y	1
$60^o + A$	$(x-\sqrt{3}y)$	$(x\sqrt{3}+y)$	2
$60^o - A$	$(x+\sqrt{3}y)$	$(x\sqrt{3}-y)$	2
2A	$(x^2 - y^2)$	2xy	1
A	x	y	1
3A	$(x^3 - 3xy^2)$	$(3x^2.y - y^3)$	1

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For L.H.S.

$$\sin A \sin(60-A) \sin(60+A)$$

$$= y \left(\frac{\sqrt{3}.x-y}{2}\right)\left(\frac{\sqrt{3}.x+y}{2}\right)$$

$$= \frac{3x^2 y - y^3}{4} = \frac{1}{4}\sin 3A \quad \text{R.H.S.}$$

4.3 **Trigonometrical Equations:**

Example 15: Solve the following equations in Tribhujank form :

(i) $\cos A = 1/\sqrt{2}$

(ii) $\cot B = 1/\sqrt{3}$

(iii) $\cot A + \tan A = 2\sec A$

Solutions :

(i) If $\cos A = 1/\sqrt{2}$,

then Tribhujank for A is (A) 1, - , $\sqrt{2}$

Therefore, 1, 1, $\sqrt{2}$, and 1, -1, $\sqrt{2}$ are two solutions.

(ii) If $\cot B = 1/\sqrt{3}$,

then Tribhujank for B is (B) 1, $\sqrt{3}$, -

Therefore, 1, $\sqrt{3}$, 2, and 1, $\sqrt{3}$, -2 are the two solutions.

(iii) Let the general Tribhujank for

A) be x , y, 1 .

Then.

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cosA + tanA = 2secA

$$\frac{x}{y} + \frac{y}{x} = \frac{2}{y}$$

$$\frac{x^2 + y^2}{xy} = \frac{2}{y}$$

$$x = 1/2$$

Therefore the Tribhujank for

A) 1 - 2

1, $\pm\sqrt{3}$, 2 are the two solutions.

5. **BEEJANK:**

In Vedic Mathematics Beejganit is the original name for Algebra. Beej essentially means the seed and ganit means mathematics.Therefore, Beejganit should essentially be the seed of different facets of Mathematics.This is quite clear and obvious if one studies Mathematics the Ancient Vedic way. Now we shall introduce ourselves to the Beejank of different Tribhujanks, that is the seed numbers. We shall study Beejank through the use of Beejganit (Algebra), although the whole thing can also be introduced through Vedic geometry.

Let p and q be the first and second elements of general Tribhujank, representing a half angle A/2.

(A/2) p, q, -

p, q, $\sqrt{p^2 + q^2}$

Therefore

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(A) $p^2 - q^2$, $2pq$, $p^2 + q^2$ (1)

Here all the elements of Tribhujanks for (A) shall be rational if p, q are rational. Further, p and q can be used to generate a perfect Tribhujank of (A) (with three elements of (A)) as such p, q are called as the Beejanks, (seed numbers)of Tribhujank of angle (A). Frequently q may be equal to unity and in that case only Beejank p is sufficient to completely define the Tribhujank.

(A) $p^2 - 1$, $2p$, $p^2 + 1$

If we substitute p equal to 3, 4, 5, 6, 7 etc. we get the following Tribhujank.

TABLE I

TRIBHUJANK

Beejank	Raw	Final	Middle element
3	3^2-1, 2x3, 3^2+1	4, 3, 4	3
	8 , 6, 10		
4	15, 8, 17	15, 8, 17	2 x 4
5	24, 10, 26	12, 5, 13	5
6	35, 12, 37	35, 12, 37	2 x 6
7	48, 14, 50	24, 7, 25	7
8	63, 16, 65	63, 16, 65	2 x 8
9	80, 18, 82	40, 9, 41	9
	etc.		

Therefore we can see that a long list of Tribhujank can be

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built up by using Beejank p only. In general, we use two Beejanks p, q. In this case since q is 1 we are adding and subtracting the square of q (which is equal to 1) to obtain the first and third element using the Vedic sutra No.7. The middle element is obtained by using the upsutra Dwandwa yoga (no. 14).(Duplex of p, q is 2pq, the middle element).

5.1 **Tribhujank Pattern:**

If we carefully study the pattern of the Tribhujanks being built up by using the whole number Beejanks from 3 onwards, we can clearly observe that we are getting two distinct groups of Tribhujanks.

Group 1: It consists of the Tribhujanks obtained from even Beejank (4;6;8 etc.) and they have following distinct properties

i) The middle element is double the Beejank (2*4 = 8; 12; 16 etc.).

ii) The third element is always 2 more than the first (15 + 2 = 17; 35 + 2 = 37 etc.).

iii) Further, two times the sum of the first and the third element is equal to the square of the middle element.

2(15 + 17) = 64 = 8*8

2(35 + 37) = 144 = 12*12 etc.

Group 2: The Tribhujanks computed by using the odd number Beejanks (3; 5; 7 etc.) and the three elements of this group exhibit the following properties:

i) The middle element is the Beejank (3;5;7 etc.)

ii) The third element is always one more than the first (4+1 = 5; 12 + 1 = 13 etc.).

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iii) Further, the sum of first and the third element is equal to the square of the middle element.

(4 + 5) = 9 = 3*3

(12 + 13) = 25 = 5*5 etc.

OR

In other words we can say that both the groups have very similar properties except that we have the factor of 2 present for even Beejank and this factor becomes 1 for odd Beejanks. The reason of this is quite obvious if we carefully study the raw and the final elements in the Table I, of Tribhujank. (as we have already divided by 2 the raw elements of odd Beejank).

These properties of Tribhujanks are very useful and can be used in two alternative ways. Firstly we can build up the three elements of any Tribhujank from the Beejank.Secondly, we can check the correctness of three elements,computed by using the Beejank formulae

$(p^2 -1, 2p, p^2 + 1)$.

It will be interesting to point out that Vedic Maths provides two alternative approaches which are independent of each other.

Example 16: Compute the Tribhujank whose Beejank is (i) 7 (ii) 13 (iii) 27 (iv) 36 (v) 31

Solutions :

(i) p = 7

Tribhujank 7*7-1, 2*7*1, 7*7+1

48, 14, 50

Therefore 24, 7, 25

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Check: (i) 25 - 24 = 1

(ii) 7*7 = 49 = 24 + 25 Okay.

(ii) p = 13

Tribhujank 13*13-1, 2*13*1, 13*13+1

84, 13, 85

Check: (i) 85 - 84 = 1

(ii) 13*13 = 169 = 84 + 85 Okay.

(iii) p = 27

Tribhujank 27*27-1, 2*27*1, 27*27+1

364, 27, 365

Check: (i) 365 - 364 = 1

(ii) 27*27 = 729 = 364 + 365

R
510
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(iv) p = 36

Tribhujank 36*36-1 , 2*36*1, 36*36+1

1295, 72, 1297

Check: (i) 1297-1295 = 2 because Beejank is even.

(ii) (72*72)/2 = 5184/2 = 2592

= 1295 + 1297 Okay.

(v) p = 31

Tribhujank 31*31-1, 2*31*1, 31*31+1

960, 62, 962

Therefore 480, 31, 481

Check: (i) 481 - 480 = 1

(ii) 31*31 = 961 = 480 + 481 Okay.

5.2 **Beejank as Tribhujank:**

It is very useful to learn that the Beejanks themselves are also the Tribhujank of half angle. If p is the only Beejank the

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corresponding Tribhujank is p, 1, - or p, 1, $\sqrt{1 + p^2}$.

Further if p, q are the Beejanks the Tribhujank is p, q, - or

p , q, $\sqrt{p^2 + q^2}$

5.3 **Two Beejanks:**

In computing the Tribhujank of table I, we have used only integer Beejanks starting with 3, we can also have a set of Tribhujanks if we take Beejank as 2.5, 3.5 etc. or in other words Beejank is 2.5, 1.

OR 5, 2 here p = 5 and q = 2. This will give as another set of Tribhujanks. In addition to those already obtained in Table I. However, the general formulae (1) for computing the Tribhujank for the given Beejanks remains the same.

Example 17: Compute the Tribhujank if (i) p, q = 12, 5 (ii) p, q = 7,2 (iii) p, q = 9,2.

Solution :

(i) p, q = 12, 5

Tribhujank 12 *12 -5*5, 2*12*5, 12*12+5*5

119, 120, 169

(ii) p, q = 7,2

Tribhujank 7*7-2*2, 2*7*2, 7*7+2*2

+45, 28, 53

(iii) p, q = 9,2

Tribhujank 9*9-2*2, 2*9*2, 9*9+2*2

77, 36, 85

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5.4 **Computing Beejank from Tribhujank:**

The Beejank of any Tribhujank can be easily obtained by using any of the two alternative methods.

(i) Since we know that Beejank itself is a Tribhujank of half angle, therefore using the half angle formulae of Tribhujank p shall be the sum of the outer two elements and q shall be the middle element itself.

Example 18: For Tribhujank 4, 3, 5 Beejank (p, q) shall be 4 + 5, 3 i.e. 9, 3; 3, 1.

(ii) The Beejank can also be obtained by using Vedic sutra(No.7),'by addition and subtraction'.p shall be the square root of the addition of the first and the third element of Tribhujank and q shall be the square root of the difference of the first and the third element of Tribhujank.

Example 19: (i) For Tribhujank 4, 3, 5

Beejank (p, q) shall be $\sqrt{5 + 4}$, $\sqrt{5 - 4}$.i.e. 3, 1

(ii) For Tribhujank 77, 36, 85

By first method 85 + 77, 36

i.e. 162, 36; 9, 2

(iii) For Tribhujank 399, 40, 401

By first method 401 + 399, 40

i.e. 800, 40; 20, 1

(iv) For Tribhujank 45, 28, 53

By first method 53 + 45, 28

i.e. 98, 28; 7, 2

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(v) For Tribhujank 480, 31, 481

By first method 480 + 481 , 31

i.e. 961, 31; 31,1

Discussion :

(i) We can see that the Beejank for any Tribhujank has much smaller numbers. As such it is always much more convenient to operate the Beejanks.

(ii) As usual Vedic Maths provides us two alternative ways and hence the second method can be used for checking.

5.5 Beejank of Beejank:

As demonstrated in the Katyayan's sulbha sutra it is quite conveniently possible to find out the Beejank of the Beejanks themselves, as Beejank itself is expressing a Tribhujank.

Example 20: The Tribhujank 119, 120, 169 has Beejank 12, 5.This itself represents a Tribhujank 12, 5, 13 as such the Beejank for 12, 5, 13 shall be 12 + 13, 5 i.e. 5, 1.

5.6 Beejank for Supplementry Tribhujank:

The Beejank for supplementary Tribhujank is obtained directly by using the Paravartaya Yojyet sutra (No. 4) i.e. by transposing the Beejank.

Example 21: If p, q = 12, 5

By transposition p, q(ST) = 5,12

5.7 **Beejank for Complementary Tribhujank** are obtained by using the sutra Sankalana Vyavakalana (No. 7) meaning,'by addition and subtraction'.

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For Beejank p, q(CT) = (p + q), (p - q)

Example 22: If p, q = 12,5

p,q(CT) = (12+5), (12-5)= 17,7

5.8 **Beejank for 0°, 90°, 120° and 270°:**

The Beejank for these shall be 1,0; 1,1; 0,1; 1, -1 or ∞,1,0,-1 respectively.Further, we can clearly see that by simply looking at the magnitude and sign of Beejank p itself we can find out the quadrant of the angle whose Beejank is represented by p.

1st quadrant From 0° to 90° Beejank p varies from ∞ to 1.
2nd quadrant From 90° to 180° Beejank p varies from 1 to 0.
3rd quadrant From 180° to 270° Beejank p varies from 0 to -1.
4th quadrant From 270° to 360° Beejank p varies from -1 to $-\infty$.

5.9 **Beejank Operations:**

As the pair of Beejanks essentially represents a Tribhujank of half angle, therefore we can add and subtract Beejanks by using the Tribhujank procedure and further the sum of the Beejanks shall also be the Beejank for the sum of the Tribhujank.This property greatly simplifies the addition and subtaction process of Tribhujank as Beejank are much smaller in magnitude.

Example 23: Beejank of (A) 12, 5, 13 shall be 5,1 and Beejank of (B) 4,3,5 shall be 3,1. Addition of Beejanks gives

p(A)	5, 1
p(B)	3, 1
p(A)+p(B)	14, 8 = 7,4

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Addition of Tribhujank

(A)	12	5	13
(B)	4	3	5
A + B	33	56	65

Beejank p(A+B) = 33 + 65, 56

i.e. 98, 56; 7,4

We can clearly see how convenient it is to add the Beejanks.

6. **ANGLES, BEEJANK AND TRIBHUJANK:**

The Beejank provide an important link between Tribhujank and their angles. If a and b are the Beejank of two angles (A) and (B) we can easily see that

p(A+B) = (ab - 1), (b + a) and

p(A-B) = (ab + 1), (b - a)

Therefore, we can easily see the following relations between the different Beejanks.

1 - 2 = 3 where, 1, 2, 3 are the Beejanks. In other words this equation represents that if we subtract the Tribhujank with Beejank 2 from the Tribhujank with Beejank 1, we shall get the Tribhujank with Beejank 3. These equations can be rearranged like ordinary equations. Therefore we can also say that 1 - 3 = 2 and 2 + 3 = 1 but 4 + 6 is not equal to 2. Further, 3 - 4 =13; 2 - 7 =3; 4- 3 = 21 - 8; 1 - 2 = 5 + 8; 2 - 3 = 12 + 17; 3-4 = 23 + 30.

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If p is the Beejank and (A) is the angle it represents (in radians) then it can be easily proved that the product p*A is approximately equal to 2. This product gets closer to 2 as p increase. (Appendix B for its derivation and accuracy).

$$p*A = 2$$

Example 24: Find the angle with Tribhujank 899, 60, 901.The Beejank is 30.Therefore the angle is 2/30 = .0666.

Example 25: What is the Tribhujank which contains an angle of 0.05.

$$p = 2/.05 = 40$$

Therefore Tribhujank is

1599, 80, 1601.

The values are accurate upto:

3 decimal places for $p \geqslant 13$

4 decimal places for $p \geqslant 24$

5 decimal places for $p \geqslant 52$

Using this simple relationship between the angle and its Beejank we can also add the column representing angle (A) in radians in the Table -I of different Tribhujank and their Beejank.Further, the error due to the use of the simple relation can also be added. These values are presented in Table -II.

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Table -II Tribhujank and Angles

TRIBHUJANK	Beejank		Angle A (radians)	Error $\frac{2}{p} - A$
0	1	1	1.5708	-
3	2*2	5	.9273	-
4	3	5	.6435	.0232
15	4*2	17	.4900	.0100
12	5	13	.3948	.0052
35	6*2	37	.3303	.0030
24	7	25	.2838	.0019
63	8*2	65	.2487	.0013
40	9	41	.2213	.0009
99	10*2	101	.1993	.0007
60	11	61	.1813	.0005
143	12*2	145	.1663	.0004
84	13	85	.1535	.0003
195	14*2	197	.1426	.0002
112	15	113	.1331	.0002
255	16*2	257	.1248	.0002
144	17	145	.1175	.0001
323	18*2	325	.1110	.0001
180	19	181	.1052	.0001
399	20*2	401	.0999	.0001
220	21	221	.0952	.0001
483	22*2	485	.0908	.0001
264	23	265	.0869	.0001
575	24*2	577	.0833	.0000
312	25	313	.0800	0
675	26*2	677	.0769	0
364	27	365	.0740	0
783	28*2	785	.0714	0
420	29	421	.0689	0
899	30*2	901	.0666	0

Computing angle from Beejank : Using Table -II we can very conveniently compute the angle represented by the given Beejank.

Example 26: Find the angle corresponding to Beejanks (i) 7,3 (ii) 10,3.

(i) This Beejank is close to the Beejank 3. Therefore the Beejank for the difference can be obtained as

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```
                 7          3
                 3          1
              ----------------
Subtracting 24          2

   or        12          1
              ----------------
```

Therefore angle (A) = 3 + 12

Using Table II(A) = .6435 +.1663

= .8098

Alternatively, this is more close to Beejank 2, therefore

```
               7          3

               2          1
            ---------------
Subtracting 17         -1
            ---------------
```

This shows the angle 2, -1 is bigger than 7,3.

Therefore angle (A) = 2 - 17

= .9273 - .1175

= .8098

(ii) This Beejank is close to the Beejank 4.

```
Therefore            10         3

                      4         1
                     -------------
       Subtracting   43         2
                     -------------
```

Therefore angle (A) = 4 + 2/(43/2)

= 4 + 4/43

= .4900 +.0930

= .5830

Since the value for 43/2 is not available in the table, we use the approximate relation p*A =2.

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Alternatively, this is more close to 3,1. Hence

```
                 10        3
                  3        1
             ------------------
Subtracting      33       -1
             ------------------
```

Therefore angle (A) = 3 - 33

= .6435 - 2/33

= .6435 -.0606

= .5829 which is slightly more accurate.

NOTES:

1. The above example clearly demonstrates that to minimize the use of table we should look for the Beejank which is closest to the ratio of the given Beejanks.So that the Beejank for the difference of the two is very large and hence the error is minimized.

2. The division of fraction (like 2/33 etc.) can be very conveniently performed straight away in one link using the Ekadhiken sutra (No.1) or Dhwajank upsutra (no. 16) explained elsewhere (Ref. 1,3,4 and 7).

3. Furhter,it is possible to very conveniently establish some useful relationships between different Beejanks. These Beejanks relations can be used for computing the angles represented by a general pair of Beejank.The following relations can be easily establish.

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2 + 2 = 3,4	3 + 3 = 4,3	4 + 4 = 15,8
5 + 5 = 12,5	3 + 5 = 7,4	2 - 5 = 11,3
1 - 4 = 5,3	2 - 4 = 9,2	1 - 7 = 4,3
1 - 5 = 3,2	2 + 8 = 3,2	2 - 12 = 5,2
3 + 17 = 5,2	3 + 12 = 7,3	3 + 27 = 8,3
3- 33= 10,3	3 - 23 = 7,2	4 + 30 = 7,2

etc.

4. We have already seen that p(-A)= -p(A).This means if we are given a Beejank say -5 then the angle is minus the angle with Beejank +5 i.e. .3948.

5. If the Beejank is large we can just divide it into 2.

6. For other Beejanks we subtract from Beejanks having a similar ratio "for which we know the angles".

7. If angle is greater than Phi/4, we can use the Tribhujank for supplementary or complementary angles.

Angles to Beejanks : This is the inverse process. We first choose an angle with known Beejank and close to the given angle. The difference of these two angles is converted to Beejank by using the formula pA =2. Finally the Beejank is obtained by adding the two Beejanks.Further, the accuracy of the result can also be obtained as $a = 2x/p^2$ (Refer Appendix B). Where p is the quantity left off p in order to make it a whole number.

Example 27: Find p for 1 radian.

1 - 0.9273 = 0.0727 using relation

pA = 2; 2/.0727 = 27.51 = 27.5 (say)

Beejank for .9273 is 2 therefore

p(1) = 2 + 27.5 = 54, 29.5 = 108, 59

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Example 28: Find p for .439 radians.

From table we know that Beejank 5 represents angle 0.3948. The difference is 0.439 - 0.3948 = 0.0442.

For the remaining angle p = 2/0.0442

= 45.25 = 45 (say)

Therefore p(.439) = 5 + 45 = 224.50

= 112,25

Accuracy a = (2*0.25)/45*45 = 0.302

Angle in Degrees : If the angles are in degrees then instead of computing p= 2/d (where d is the difference) we compute 114.59/d as 2*Phi = 360.

Example 29: Find Beejank for angle of 58.58.

We know p(53.13) = 2

58.58- 53.13 =5.45

p(5.45) = 114.59/5.45 = 21 .Therefore

p(58.58) = 2 + 21 = 41,23

Example 30: Solve cosA+ 7sinA = 5.

If p is the Beejank of A then we know that the Tribhujank of A shall be $p^2 - 1$, 2p , $p^2 + 1$. Therefore

$$\sin A = 2p/(p^2 +1); \cos A = (p^2 -1)/(p^2 +1)$$

Therefore given equation is

$$(p^2 - 1)/(p^2 + 1) + (7 *2p)/(p^2 + 1) = 5$$

$$p^2 - 1 + 14p = 5(p^2 +1)$$

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or $2p^2 - 7p + 3 = 0$ by using Vedic upsutra Adyam (No. 3)

$(2p - 1)(p - 3)$

$\therefore \quad p = 1/2, 3$

Hence $\quad A = \text{Phi} - 0.9273, 0.6435$

$= 2.2143, 0.6435$

Example 31: Solve 3cosA + 2sinA =3.

Therefore $3p^2 - 3 + 4p = 3p^2 + 3$

$\therefore \quad p = \infty, 1.5$

Hence $\quad A = 0, 1.5708 - 0.3948$

$= 0, 1.1760$

As p =1.5 = 3,2 which is equal to 1- 5.

CONCLUSIONS:

i) The Vedic Tribhujank concepts provide a very convenient system for handling wide class of problems including geometry (2-D and 3-D), Trigonometry (Plane and Spherical) Transformations and Astronomical computations.

ii) The number of formulae required for working with Vedic maths system is much lesser and procedures are simpler compare to the present methods.

iii) Over all there is tremendous simplicity and ease introduced by Vedic procedure.

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APPENDIX-A

GEOMETRICAL PROOF:

Given a Tribhujank (A) x,y,r draw a circle with centre O and radius r, as shown in Fig. 10 & 11. Produce QO to R, and join RP. Then R = A/2, so that the Tribhujank for A/2 is (A/2) x + r, y, -----.

Further, let us draw triangle (A/2) 3,1, $\sqrt{10}$. Then by doubling the angle we have (A) 8, 6, 10 or 4,3,5. (See Fig. 13).

ALGEBRAIC FORMULATION:

Consider the Beejank of half angle (A/2) p,q. By doubling this we get (A).

A/2	p,	q,	-
A/2	p,	q,	$\sqrt{p^2+q^2}$
A	p^2-q^2,	2pq,	$p^2 + q^2$

APPENDIX - a

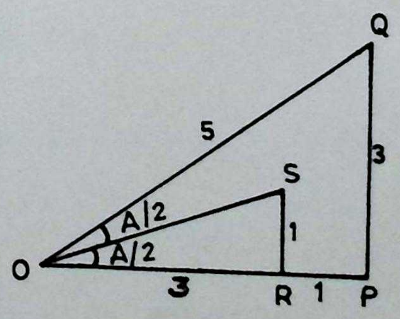

FIG. 13

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APPENDIX-B

A NOTE ON ACCURACY:

In the conversion of Beejank into angles. If we assume that the known angles (from Table II) are known to whatever degree of accuracy is required, the only inaccuracy arising is due to the use of the approximation pA = 2.

From equation p = cot A/2, we get

$$A = 2\tan^{-1}(1/p)$$

and using the series expansion for $\tan^{-1} x$

$$A = 2/p - 2/3p^3 + 2/5p^5 - 2/7p^7 + -----$$

Neglecting terms with p^3 and higherr powers we have A = 2/p or pA = 2 as derived before.

But since the next term, in the series for A above, is $2/3p^3$, this can be used either for a mere precise value of A, or as an estimate of the inaccuracy in using 2/p for A.

For accuracy to 4 decimal places we require $2/3p^3 < 0.00005$, from which p > 24.

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SPIRITUAL STUDY GROUP
(SSG)

आध्यात्मिक अध्ययन संस्था
(आऽस)

★ॐ श्री : ★

VEDIC MATHEMATICS
OR
SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS
Maths with Smiles

Calculas

वेदात्सर्वं प्रसिद्ध्यति ।
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
Vedas are the Eternal Source of Knowledge.

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CALCULUS

1.0 DIFFERENTIATION:

The differentiation of a function is obtained simply by applying the Ekanunena Purvena Sutra (No. 14) which means one less than the previous one.

1.1 PROCEDURE:

We multiply the function by the power of the variable and reduce the power by 1.

Example 1: x^4

Differential of $x^4 = 4x^{4-1} = 4x^3$ (the first derivative)

$D_1 = 4x^3$

The same process is repeated for subsequent derivatives.

Second derivative, $D_2 = 4 * 3\ x^{3-1} = 12\ x^2$

Third derivative, $D_3 = 4 * 3 * 2\ x^{2-1} = 24\ x$

Fourth derivative, $D_4 = 24 * 2\ x^{1-1} = 48\ x^0 = 48$

Fifth derivative, $D_5 = 48 * 0\ x^{0-1} = 0$

1.2 Differentiation of sinx = cosx and of cosx = -sinx . This can be clearly explained and understood by using the concept of त्रिभुजांक (Tribhujank) (Ref. 5). Further, if x has been multiplied by some constant term, the same shall also appear outside using Upsutra 'Anurupyena' (No.1) - Proportionately.

Example 2: $D = \sin 2x$

$D_1 = 2 \cos 2x$

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Similarly, $D_2 = 2*2(-\sin 2x) = -4 \sin 2x$ etc.

Example 3: $D = \cos 3x$

$D_1 = 3\ (-\sin 3x) = -\ 3 \sin 3x$

$D_2 = -3*3 \cos 3x = -9 \cos 3x$ etc.

1.3 Differential of e^x is the same e^x. However, if x is multiplied by some constant, the differential is also multiplied by the same constant using the Upsutra Anurupyena (No. 1).

Example 4: $D = e^{2x}$

$D_1 = 2\ e^{2x}$

$D_2 = 2 * 2\ e^{2x} = 4\ e^{2x}$ etc.

Example 5: $D = e^{-3x}$

$D_1 = -\ 3\ e^{-3x}$

$D_2 = (-3)(-3)\ e^{-3x} = 9\ e^{-3x}$ etc.

1.4 Differential of log x is $\frac{1}{x}$. Further, if x is multiplied by any constant then by 'Anurupyena'(No. 1), we also multiply the differential by the same constant.

Example 6: $D = \log 3x$

$$D = \frac{3*1}{3x} = \frac{1}{x}$$

However, the further derivatives can be obtained by considering

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$\frac{1}{x} = x^{-1}$; as usual.

Discussion: It is interesting to observe that the first differential of logax, where a is a constant, shall always be $\frac{1}{x}$.

1.5 Differential of sum of two functions :

Example 7: $D = x^2 + 3\sin x$

The differential of each function shall be obtained separately and the algebraic addition of these differentials shall be the desired differential of the total function.

$$D_1 = 2x + 3\cos x$$

$$D_2 = 2 + 3(-\sin x)$$

$$= 2 - 3\sin x. \quad \text{etc.}$$

Example 8: $D = e^x + \log 4x - 2\cos x$

$$D_1 = e^x + \frac{4*1}{4x} - 2(-\sin x)$$

$$= e^x + \frac{1}{x} + 2\sin x.$$

1.6 Differential of product of two functions :-

The different order derivatives can be obtained directly by using the Vedic Urdhava Sutra (Ref. 1,2,3).

Example 9: Find the first derivative of $x^6 e^{3x}$.

For computing the first derivative we shall write the two parts of the given function x^6 & e^{3x} in vertical column and write their respective derivative in the subsequent column. The product

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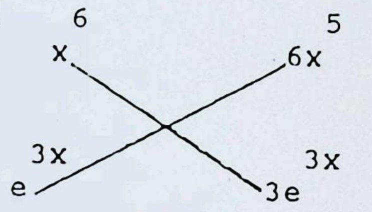
itself is obtained by the vertical simplification of the first column.

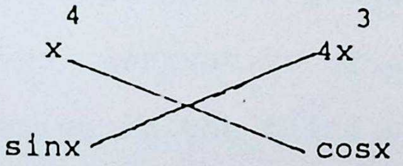
Therefore $D = x^6 . e^{3x}$

The first derivative is obtained by the cross multiplication.



Therefore $D_1 = x^6 .3e^{3x} + e^{3x} .6x^5$ (crosswise)

Example 10: x^4 sinx.



$D = x^4 . \sin x$ (vertically)

$D_1 = x^4 . \cos x + 4x^3 \sin x$. (crosswise)

1.6.1 Higher Derivatives:

We have seen how convenient it is to find out the different powers of numbers by using the Vedic Upsutra Anurupyena (5).In this process we make use of the multipliers which are the coefficients of different powers of 11. It is interesting to mention that the same factors also come into play while finding the different derivatives

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of the product of functions. The coefficients are as under :

11^0 = 1

11^1 = 1 1

11^2 = 1 2 1

11^3 = 1 3 3 1

11^4 = 1 4 6 4 1

.

.

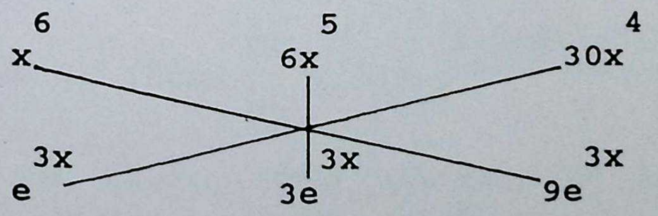
etc.

The subsequent terms in higher order rows are obtained by the addition of the adjacent digits in the previous row. For the first derivative we have used the coefficients of the second row, namely 1,1.

Similarly, if we are interested to compute the higher derivatives we shall be using the corresponding coefficients.

For second derivative of a product we shall also write the second derivative in the third column in addition to the first and second column.Further,we simplify the Urdhava way, multiplying the corner terms crosswise and the central terms vertically, as usual. The three products shall ofcourse be multiplied by the corresponding coefficients of the third row. (11^2); namely 1, 2 & 1.

Example 11: $x^6 . e^{3x}$



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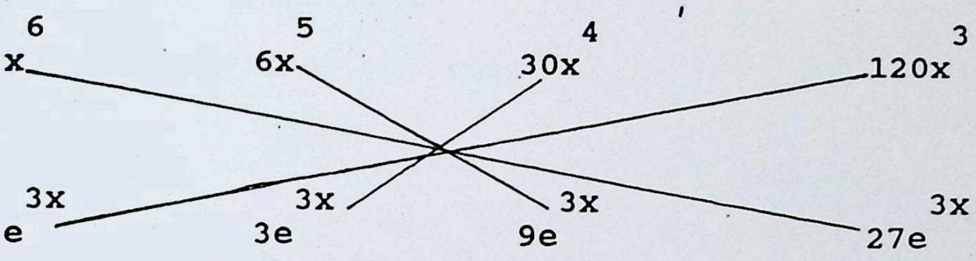
Therefore,

$$D_2 = 1*x^6*9e^{3x} + 2*6x^5*3e^{3x} + 1*30x^4 e^{3x}$$

$$= 9x^6 e^{3x} + 36x^5 e^{3x} + 30\,x^4 e^{3x}$$

Third Derivative: The four terms are multiplied by the four coefficients 1, 3, 3 & 1.



Therefore,

$$D_3 = 1*x^6*27e^{3x} + 3*6x^5*9e^{3x} + 3*30x^4 * 3e^{3x} + 1*120x^3*e^{3x}$$

$$= 27x^6 e^{3x} + 162x^5 e^{3x} + 270x^4 e^{3x} + 120x^3 e^{3x}$$

Example 12: $\frac{dy}{dx} = \lim_{x\to 0} \frac{y}{x}$ This is a definition of $\frac{dy}{dx}$. It expresses the last or ultimate ratio of two vanishing quantities.

If we have some function of x, f(x), or just f, then $\frac{df}{dx} = \lim_{\delta x\to 0} \frac{\delta f}{\delta x}$.

To get δf, the change in f(x) corresponding to a small change, δx, in x we use the Vedic sutra "By Addition and By Subtraction". We increase x by δx, and subtract the two values of f corresponding to $x + \delta x$ and x. Thus $\frac{df}{dx} = \lim_{\delta x\to 0} \frac{f(x+\delta x) - f(x)}{x}$. This is a fundamental equation for finding derivatives, and in addition to using sutra (No.7)"By Addition and by subtraction" also makes use of two other

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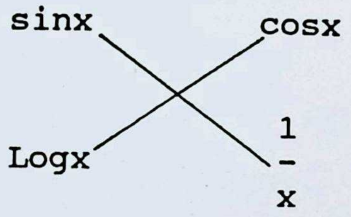
Vedic formulae - (sutra No. 6) "One in Ratio-the Other One Zero" and "Only the last terms" (upsutra No. 9). Having found the change in f(x) using "By Addition and By Subtraction", we put it in ratio with x, which we allow to tend to zero.Then "Only the Last terms" (upsutra No.9) gives us the final limit of the ratio, which is $\frac{df}{dx}$.

Example 13: $D = \sin x.\log x.$

We write the columns as usual.



$D = \sin x.\log x$ (vertically)

$D_1 = \sin x.\frac{1}{x} + \cos x.\log x$ (crosswise)

Similarly the higher derivatives can also be obtained.

1.7 Derivative of a function consisting of the product of 3 terms

It is interesting to point out that even the derivative of the product of three functions can be similarly obtained by using the vertically crosswise method for multiplying three numbers (Ref. 5).

URDHAVA METHOD

In case we have to multiply three numbers of two digits each, the second part of the answer is obtained by cross multiplication of two columns.

$R_2 = U_1 \times M_1 \times L_2 + U_1 \times L_1 \times M_2 + M_1 \times L_1 \times U_2$

where U_1 & U_2 are the first and second (from left) terms in the

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upper row. Similarly, M_1, M_2 and L_1, L_2 are the terms of the middle and the lower rows

Example 14: $D = x^2 . e^{3x} . \sin x$

We write the first and second column of three terms. The second column shall contain the first derivative of each term.

$$\begin{array}{ll} x^2 & 2x \\ e^{3x} & 3e^{3x} \\ \sin x & \cos x \end{array}$$

By using the first two steps of multiplication by the Urdhava method from left to right.

$$D = x^2 . e^{3x} \sin x \quad \text{(vetically)}$$

$$D_1 = x^2 e^{3x} \cos x + x^2 \sin x \, 3e^{3x} + 2x \, e^{3x} \sin x \quad \text{(crosswise)}$$

$$= x^2 e^{3x} \cos x + 3x^2 \sin x \, e^{3x} + 2x \, e^{3x} \sin x$$

The result is obtained directly in a single step and no unnecessary working is required to be done.

1.7.1 Second Derivative

If we are interested to further compute the second derivative of this function consisting of three parts.

The three columns are written as before.

$$\begin{array}{lll} x^2 & 2x & 2 \\ e^{3x} & 3e^{3x} & 9e^{3x} \\ \sin x & \cos x & -\sin x \end{array}$$

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$$D_2 = -x^2 e^{3x} \sin x + 9x^2 \sin x\, e^{3x} + 2e^{3x} \sin x + 2*x^2\, 3e^{3x} \cos x$$

$$+ 2*2x\, e^{3x} \cos x + 2*2x \,.\, 3e^{3x} \sin x$$

We double the terms of vertical simplification of the middle column pairs as before.

Finally:

$$D_2 = 8x^2 e^{3x} \sin x + 6x^2 e^{3x} \cos x + 4xe^{3x} \cos x + 12xe^{3x} \sin x + 2e^{3x} \sin x$$

is the 2nd derivative of the function consisting of the product of the 3 terms, obtained directly in one line working. As per present system of working by parts we shall require more than one page of working. Further this method can be extended to compute the higher derivatives by the direct application of the Urdhava sutra.

1.8 Differentiation of a function consisting of ratio of two variables

We know that $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$$

It is interesting to observe that the differential of a ratio of two polynomials can be directly obtained in one line by the simple application of the Urdhava sutra (No.3). The only procedural difference is that we find the difference of the cross product of the coefficients of different powers of x. This is quite similar to the procedure used in solving the set of simultaneous equation (Refer 5, 6 & 7).

Example 15: Differentiate $z = \dfrac{3 + 7x}{1 + 3x}$

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$$\frac{dz}{dx} = \frac{(1*7 - 3*3)(1 - 0)}{(1 + 3x)^2} = \frac{-2}{(1 + 3x)^2}$$

where the second bracket contains the difference of the powers of x in the two terms being cross multiplied. Further the vertical multiplication does not appear in the numerator as the difference of the powers of x would be zero.

Example 16: Differentiate $z = \dfrac{2 + 3x + 5x^2}{3 + 4x + 7x^2}$

$$\frac{dz}{dx} = \frac{(3*3-2*4)(1-0) + (3*5-7*2)(2-0)x + (4*5-3*7)(2-1)x^2}{(3 + 4x + 7x^2)^2}$$

$$= \frac{1 + 2x - x^2}{(3 + 4x + 7x^2)^2}$$

The powers of x in different terms are equal to one less than the sum of the powers of different terms being cross multiplied. This is in accordance with the Ekanyunena sutra (No.14) used f differentiation. Similarly, it is possible to find out the differential of ratio of even bigger polynomials.

Example 17: Differentiate $z = \dfrac{1 + 4x + 2x^2 + 7x^3}{2 + 3x + 5x^2 + 11x^3}$

As already explained the numerator is obtained by using the Urdhava sutra.

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$(2*4-1*3)(1-0) + (2*2-1*5)(2-0)x + (2*7-1*11)(3-0)x^2$
$+ (3*2-4*5)(2-1)x^2 + (3*7-4*11)(3-1)x^3 + (5*7-2*11)(3-2)x^4$

$= 5 - 2x + 9x^2 - 14x^2 - 46x^3 + 13x^4$

Therefore

$$\frac{dz}{dx} = \frac{5 - 2x - 5x^2 - 46x^3 + 13x^4}{(2 + 3x + 5x^2 + 11x^3)^2}$$

DISCUSSIONS:

1. We have seen how convenient and easy it has become to obtain the differential of the ratio of two polynomials by using the Urdhava sutra.

2. It is interesting to mention that the difficult task of finding the differential of the numerator and denominator separately, multiplying and then simplification is drastically simplified.

3. In case any particular intermediate term is missing it will be convenient to add the missing term with zero coefficient.

2.0 **INTEGRATION**

A simple function like x^n can be integrated directly by using the very first Vedic sutra Ekadhiken Purven (एकाधिकेन पूर्वेण) which means by one more than the previous one.

Ekadhiken of n is n + 1 therefore integral of $x^n = \frac{x^{n+1}}{n + 1} + c$

We note that this is converse to differentiation,we also divide by (n + 1) and add constant c as the differential of the constant c is zero.

Example 18: $I = \int x^5 \, dx$

Ekadhiken of 5 is 6. Therefore

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$$I = \frac{x^6}{6} + c$$

2.1 Integration of cosx = sinx and

of sinx =-cosx. This can be easily explained by using the concept of Tribhujank(त्रिभुजांक) (Ref. 5).

Further, if x is multiplied by the constant the integrated function shall be divided by the same constant using upsutra Anurupyena.

Example 19: $I = \int \cos 2x \, dx$

$$I = \frac{\sin 2x}{2} + C$$

Example 20: $I = \int \sin 3x \, dx$

$$I = -\frac{\cos 3x}{3} + C$$

2.2 The integral of $\frac{1}{x}$ is equal to log x. Similarly,

$$\int e^x \, dx = e^x$$

Example 21: $I = \int e^{ax} \, dx$

where a is a constant.

By using Anurupyena sutra (No.1)

$$I = \frac{e^{ax}}{a} + C$$

Since x is multiplied by a, the integral is divided by a.

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2.3 Integration of the product of two functions : For this, we first write the function which can be easily differentiated and write the differential of the same.The second function is writen in the second line and write its integration after it, and then apply the Urdhava sutra.The signs of the products are positive and negative alternatively.

Example 22: $I = \int x^3 \log x \, dx$

$$\begin{array}{ll|l} \text{Logx} & & \frac{1}{x} \\ & \searrow & \\ x^3 & & \frac{x^4}{4} \end{array}$$

Therefore,

$$I = \frac{x^4}{4} \log x - \int \frac{1}{x} \quad \frac{x^4}{4} \; dx$$

This can be further integrated by using the Ekadhiken sutra (No.1).

$$= \frac{x^4}{4} \log x - \frac{x^4}{16} + C$$

Example 23: $I = \int x^3 \; \sin x \;\; dx$

It will be convenient to use x^3 as the first term; as the differential of the same ultimately reduces to zero.

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$$\begin{array}{cccc} x^3 & 3x^2 & 6x & 6 \\ \sin x & -\cos x & -\sin x & \cos x \end{array}$$

Using Urdhava sutra.

Therefore,

$$I = x^3(-\cos x) - 3x^2(-\sin x) + 6x\cos x - \int 6\cos x \, dx$$

The integral can be written at sight.

$$= -x^3 \cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + C$$

This integration in general would require three repeated applications of integration by parts.

Alternatively, we can continue to differentiate and integrate until a zero is obtained and then the integral is just the crosswise multiplication of the two rows.

$$\begin{array}{ccccc} x^3 & 3x^2 & 6x & 6 & 0 \\ \sin x & -\cos x & -\sin x & \cos x & \sin x \end{array}$$

$$I = -x^3 \cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + C$$

Example 24: $I^2 = \int x^2 \cos 3x \, dx$

$$\begin{array}{cccc} x^2 & 2x & 2 & 0 \\ \cos 3x & \dfrac{\sin 3x}{3} & \dfrac{-\cos 3x}{9} & \dfrac{-\sin 3x}{27} \end{array}$$

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$$I = \frac{x^2 .\sin 3x}{3} + \frac{2x.\cos 3x}{9} - \frac{2.\sin 3x}{27} + C$$

Example 25: $I = \int e^{ax} \sin bx \, dx$

e^{ax}	ae^{ax}	$a^2 e^{ax}$
$\sin bx$	$\frac{-\cos bx}{b}$	$\frac{-\sin bx}{b^2}$

$$I = e^{ax} \left(\frac{-\cos bx}{b}\right) + \frac{a e^{ax} \sin bx}{b^2} - \int \frac{a^2 e^{ax}}{b^2} .\sin bx \, dx + A$$

transposing

$$I + \int \frac{a^2}{b^2} e^{ax} \sin bx \, dx = \frac{-e^{ax} \cos bx}{b} + \frac{a e^{ax} \sin bx}{b^2} + A$$

$$I + \frac{a^2}{b^2} I = \frac{-e^{ax} \cos bx}{b} + \frac{ae^{ax} \sin bx}{b^2} + A$$

$$I = \left(\frac{b^2}{b^2 + a^2}\right) \left[\frac{-e^{ax} \cos bx}{b} + \frac{ae^{ax} \sin bx}{b^2}\right] + A$$

$$= \frac{e^{ax}}{b^2 + a^2} \left[-b\cos bx + a\sin bx\right] + A$$

Example 26: $I = \int \frac{x^3}{1 + x^4} dx$

The example of this type can be solved directly by Vilokanam

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upsutra (No.12). In this case the numerator (x^3) is directly the differential of the denominator and we just need to multiply & divide by 4.

Therefore,

$$I = \frac{1}{4} \log (1 + x^4) + C$$

Reduction Formulae:

The integral of the multiplication functions like those in example 24 can be directly obtained in one line by using the Paravartaya yojyet sutra (No.4) following the steps as explained below.

$$I = \int e^{ax} . \sin bx \, dx$$

We integrate the exponential function and multiply with the second function (sinx). Further, we integrate the exponential function and multiply with the differential of the second function. The sign of the second term is reversed due to alteration of signs as usual. These terms are finally divided by 1 more than the squared ratio of the coefficient of x in the trig function (b) by the coefficient of x in the exponential term (a). Therefore,

$$I = A + \frac{\frac{1}{a} e^{ax} \sin bx - \frac{1}{a^2} e^{ax} . b\cos bx}{1 + (b/a)^2}$$

is the final integral, obtained in one line.

Similarly;

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Example 27: $I = \int e^{2x} \sin 3x \, dx$

$$I = A + \frac{\left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{1}{2^2} e^{2x} \, 3\cos 3x \right\}}{1 + (3/2)^2}$$

Example 28: $I = \int e^{4x} \cos 2x \, dx$

$$I = A + \frac{\left\{ \frac{1}{4} e^{4x} \cos 2x - \frac{1}{4^2} e^{4x} \, 2(-\sin 2x) \right\}}{1 + 2^2/4^2}$$

$$= A + \frac{\frac{1}{4} e^{4x} \cos 2x + \frac{1}{8} e^{4x} \sin 2x}{5/4}$$

The result can be checked by differentiating with respect to x.

Example 29: $I = \int x^2 (1 + x)^6 \, dx$

x^2	$2x$	2	0
$(1 + x)^6$	$\frac{(1+x)^7}{7}$	$\frac{(1+x)^8}{7.8}$	$\frac{(1+x)^9}{7.8.9}$

Therefore,

$$I = \frac{x^2 (1+x)^7}{7} - \frac{2x.(1+x)^8}{7.8} + \frac{2.(1+x)^9}{7.8.9} + C$$

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Example 30: $I = \int e^{-x^2/2} dx$

$$I = C + e^{-x^2/2}\left[x + \frac{x^3}{3} + \frac{x^5}{3.5} + \frac{x^7}{3.5.7} + \dots\dots \right]$$

Note that the $\frac{d}{dx}(e^{-x^2/2}) = -x\, e^{-x^2/2}$

The whole integral can be obtained directly in one line.

Example 31: $I = \int x \sin(x^3)\, dx$

$$I = A + \sin x^3 \left[\frac{x^2}{2} - \frac{3^2 . x^8}{2.5.8} + \frac{3^4 x^{14}}{2.5.8.11.14} \dots\dots \right]$$

$$+ \cos x^3 \left[\frac{-3}{2} \frac{x^5}{5} + \frac{3^3 x^{11}}{2.5.8.11} \dots\dots\dots\dots \right]$$

Integration of Rational Fractions : The integration of rational fractions can be conveniently done by splitting the given fractions into partial fractions which then can be directly integrated in the log form. In the present system computing the partial fractions takes a very long time, space and effort . But these partial fractions can be easily obtained by using the Paravartaya sutra (No.4) in one line (Refer 5), which then can be integrated in the second step.

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Example 32:

$$I = \int \frac{(x + 1)}{x^3 + x^2 + 6x}\, dx$$

$$I = \int \frac{x + 1}{x(x-2)\ (x+3)}\, dx$$

$$= \frac{-1}{6}\int \frac{dx}{x} + \frac{3}{10}\int \frac{dx}{x-2} - \frac{2}{15}\int \frac{dx}{x+3}$$

$$= \frac{-1}{6} \log x + \frac{3}{10} \log(x-2)\ \frac{-2}{15} \log(x+3)$$

Example 33:

$$I = \int \frac{x^3}{(x-1)(x-2)(x-3)}\, dx$$

$$I = \int 1\, dx + \int \frac{dx}{2\ (x-1)} - \int \frac{8dx}{(x\ -2)} + \int \frac{27dx}{2(x-3)}$$

$$= x + \frac{1}{2} \log(x-1) - 8\log(x-2)\ \ \frac{27}{2} \log(x-3)$$

3.0 NON LINEAR DIFFERENTIAL EQUATIONS:

After having an introduction to the direct and simple Vedic methods of differentiation and integration of various types of function, we can now study the solution of non linear differential, integral and integro differential equations. It is interesting to point out that the well known method of series solution when combined with the Vedic sutras can be used very efficiently to solve a wide variety of problems. In particular, this procedure is very conveniently applicable to the problems where the boundary condtions are given.

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Example 34: Solve $y' + y^2 = 1 + x^2 + 2x^4$ given that $y(o) = 0$

The variable y can easily be assumed to be equal to a power series in different powers of the second variable x.

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + \text{-----}$$

where a, b, c, d, e etc.are constants.

Therefore by Ekanyunena Purvena sutra (No. 14).

$$y' = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + \text{-----}$$

First we use the boundary conditions.

Now we compare the absolute term and then terms of different powers of x.

$$y = \overset{0}{a} + \overset{1}{b}\,x + \overset{0}{c}\,x^2 + \overset{0}{d}\,x^3 + \overset{0}{e}x^4 + \overset{2/5}{f}\,x^5 + \text{-------}$$

$$y' = \underset{1}{b} + \underset{0}{2c}x + \underset{0}{3d}x^2 + \underset{0}{4e}x^3 + \underset{2/5}{5f}x^4 + \text{-------}$$

Since $y(0) = 0$ .Therefore $a = 0$

comparing constant term, we get

$$b + a^2 = 1$$

Therefore $b = 1$.

Comparing coefficients of x, we get $2c + 2ab = 0$ as R.H.S. has no term in x. L.H.S. has y^2, as such using Dwandwa yoga upsutra (No.14) (Ref. 5,8) we get 2ab as the coefficient of x. Therefore

$$c = 0$$

comparing the coefficients of x^2, we get $3d + 2ac + b^2 = 1$. Here duplex of three terms a, b & c directly comes into the equation.

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Therefore

$$d = 0$$

Comparing the coefficients of x^3, we get $4e + 2ad + 2bc = 0$

Therefore $e = 0$

Comparing the coefficients of x^4, we get $5f + 2ae + 2bd + c^2 = 2.$

$$f = 2/5$$

Further, comparing the higher powers of x would yield the subsequent constants g & i etc. as zeros and h = -4/35 etc.

Hence

$$y = x + 2/5\, x^5 - 4/35\, x^7 \quad \text{-------}$$

Example 35: Solve $y'' + 2y^2 = 2 + 3x + x^2$

$$y(0) = o;\; y'(0) = 2$$

Using boundary conditions a = 0 and b = 2.

$$y = a^{0} + b^{2}x + c^{1}x^{2} + d^{1/2}x^{3} + e^{-7/12}x^{4} + fx^{5} + \text{----------}$$

$$y' = b^{2} + 2c^{1}x + 3d^{1/2}x^{2} + 4e^{-7/12}x^{3} + 5fx^{4} + \text{----------}$$

$$y'' = 2c_{1} + 6d_{1/2}x + 12e_{-7/12}x^{2} + 20fx^{3} + \text{-----------}.$$

Comparing the constant term, x term, x^2 term

constant term	x term	x^2 term
$2c + 2.a^2 = 2$	$6d + 2*2ab = 3$	$12e + 2*2ac + 2b^2 = 1$
$c = 1$	$d = 1/2$	$12e + 2*4 = 1$
		$e = -7/12$

We use the Dwandava yoga (upsutra No.14) for computing terms

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corresponding to $2y^2$.

$$y = 2x + x^2 + 1/2x^3 - 7/12x^4 + \text{----}$$

Example 36: $y + (y')^2 + 2y'' = \exp(x) + 2\exp(2x)$

$y(0) = 3;\ y'(0) = 2$

As we have the term $(y')^2$, we may expect 2 solutions but due to the boundary condition $y'(0) = 2$ we should have only one solution. We know that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \text{---}$$

Therefore,

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \text{-----}$$

As such,

$$e^x + 2e^{2x} = \left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\text{---}\right]+2\left[1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\frac{(2x)^4}{4!}+\text{---}\right]$$

$$= 3 + 5x + \frac{9x^2}{2!} + \frac{17x^3}{3!} + \frac{33x^4}{4!} + \text{-------}$$

Assuming

$$y = a^3 + bx^2 + c^{-1}x^2 + dx^3 + ex^4 + \text{------}$$

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Therefore

$$y' = b^2 + 2c_{-1}\, x + 3dx^2 + 4ex^3 + \text{-------}$$

and $$y'' = 2_{-1}\, c + 6dx + 12ex^2 + \text{------}.$$ (using boundary conditions $a = 3;\ b = 2$)

Constant terms	x terms	x^2 terms
$a + b^2 + 4c = 3$	$b + 4bc + 12d = 5$	$c + 4c^2 + 6bd + 24e = 9/2$
$c = -1$	$d = 11/12$	$e = -19/48$

Hence,

$$y = 3 + 2x - x^2 + \frac{11}{12} x^3 - \frac{19}{48} x^4 + \text{------}$$

Example 37: $yy' + 2y'' = \exp(x)\sin 2x$ given that

$y(0) = 1\ ,\ y''(0) = 2$

We know that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \text{-----}$$

$$\sin 2x = 2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \text{-----}$$

Therefore,

$$e^x \sin 2x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \text{--}\right)\left(2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \text{------}\right)$$

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$$= 2x + 2x^2 + \frac{2x^3}{2!} + \frac{2x^4}{3!} + \frac{2x.x^4}{4!}$$

$$+ \frac{(2x)^3}{3!} + \frac{(2x)^3.x}{3!} + \frac{(2x)^3.x^2}{3!\ 2!} + -----$$

$$+ \frac{(2x)^5}{5!} + \frac{(2x)^5.x}{5!} + ------$$

$$= 2x + 2x^2 + \frac{14x^3}{6} + \frac{5x^4}{3} + \frac{32x^5}{5} + ----$$

Assuming

$$y = \overset{1}{a} + \overset{-4}{b}x + \overset{1}{c}x^2 + dx^3 + ex^4 + -----$$

$$y' = b + 2cx + 3dx^2 + 4ex^3 + ------$$

$$y'' = 2c + 6dx + 12ex^2 + ------$$

Since $y(0) = 1$

$\therefore \quad 1 = a$

and $y''(0) = 2$

$2c = 2$; $\quad c = 1$

Constant term

$ab + 2x.2c = 0$

$b = -4$

x term

$2ac + b^2 + 6d * 2 = 2$

$d = -4/3$

x^2 term

$3ad + bc + 2bc + 24e = 2$

$e = 3/4$

$$y = 1 - 4x + x^2 - 4/3x^3 + 3/4x^4 + ----$$

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Example 38: $y + 2y'^2 = 7 - 6x + 11x^2$

if $y(1) = 4$

The boundary condition is not given at the origin, so we expand y as a power series in (x-1). Therefore given equation can also be written as

$$y + 2y'^2 = 12 + 16(x-1) + 11(x-1)^2, \text{ where } y(1) = 4$$

Assuming that

$$y = a + b(x-1) + c(x-1)^2 + d(x-1)^3 + e(x-1)^4 + f(x-1)^5 + ---$$

$$y' = b + 2c(x-1) + 3d(x-1)^2 + 4e(x-1)^3 + 5f(x-1)^4 + ----$$

Using boundary condition,

$a = 4$; Further comparing coefficients:

Absolute term

$a + 2b^2 = 12$

$4 + 2b^2 = 12$

$2b^2 = 8$

$b^2 = 4$

$b = \pm 2$

Terms of (x-1)

$b + 2*2*2cb = 16$

$b(1 + 8c) = 16$

Since b has 2 values, therefore

either $c = 7/8$

or $c = -9/8$

Terms of $(x-1)^2$

$c + 2 [2*3bd + 4c^2] = 11$

$c + 12bd + 8c^2 = 11$

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$$\frac{7}{8} + 24d + \frac{49}{8} = 1$$

$$d = 1/6$$

Similarly three more values of d shall be obtained using three remaining combination of the values of b & c. One solution is

$$y = 4 + 2(x-1) + \frac{7}{8}(x-1)^2 + \frac{1}{6}(x-1)^3 + ----$$

4.0 **INTEGRAL EQUATIONS**

Similarly we can solve the integral equation and integro differential equations.

Example 39: Solve $\int_0^x\int_0^x ydx.dx + 2\int_0^x y\,dx = \frac{x^3}{2} + 5x^2 + 8x$

Assume $y = a^4 + bx^3 + cx^{0\,2} + dx^{0\,3} + ex^4 + ----$

Integrating

$$\int_0^x ydx = a^4x + \frac{b^{3/2}}{2}x^2 + \frac{c^0}{3}x^3 + \frac{d^0}{4}x^4 + ----$$

$$\int_0^x\int_0^x ydx = \frac{a^2x^2}{2} + \frac{b^{1/2}x^3}{6} + \frac{c^0x^4}{12} + ----$$

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Coefficient of	Equation	
x	$2a = 8$	; $a = 4$
x^2	$2.\frac{b}{2} + \frac{a}{2} = 5$	; $b = 3$
x^3	$2.\frac{c}{3} + \frac{b}{6} = \frac{1}{2}$	; $c = 0$
x^4	$2.\frac{d}{4} + \frac{c}{12} = 0$	; $d = 0$

Hence exact solution is

$$y = 4 + 3x$$

Similarly we can solve even the non linear integro-differential equations.

Example 40: Solve $2y^2 + (y')^2 + y\int_0^x y\,dx = 18 + 9x + 40x^2 + 8x^3 + 8x^4 + \frac{4}{3}x^5$

where $y(0) = 3$; $y'0) = 0$

Assuming

$$y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + ---$$

$$y' = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + 7hx^6 + ---$$

& $$\int_0^x y\,dx = ax + \frac{b}{2}x^2 + \frac{c}{3}x^3 + \frac{d}{4}x^4 + \frac{e}{5}x^5 + \frac{f}{6}x^6 + ---$$

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Boundary conditions lead to a = 3 and b = 0

Coefficient	Equation
Constant	$2*a^2 + b^2 = 18$; Okay
x	$2*2ab + 2b*2c+a^2 = 9$; Okay
x^2	$2*2ac+2b^2 +2*3bd+4c^2 +ab+\frac{ab}{2} = 40$
	OR $12c+0+0+4c^2 = 40$; c= 2(OR -5)

Case 1 : Taking c = 2

x^3	$2*2ad+2*2bc +2*4be+2*6cd+\frac{ac}{3} +\frac{b^2}{2}+ac$
	= 8 OR
	12d+ 0+ 0+24d+2+0 +6= 8; OR d = 0
x^4	$2*2ae +2c^2 +2*8ce = 8$;
	12e + 8 +32e = 8 ; e = 0
x^5	$2*2af + 2*10cf +\frac{c^2}{3} = 4/3$;OR f =0

Hence one solution is $y = 3+ 2x^2$.

5.0 TRANSCENDENTAL EQUATIONS AND PARTIAL DIFFERENTIAL EQUATIONS

Even the equations of these types have been successfully solved using the Urdhava Sutra, other Vedic sutras and the series solution method.The details are available elsewhere (Ref. 10).

Example 41: $\int_0^x y\, dx + 2\left(\frac{dy}{dx}\right)^2 = 2x + 8x^2 + 7x^3 + \frac{9}{4}x^4$

If y(0) =2 & y'(0) = 0

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$$y = a^{2} + bx + cx^{2} + dx^{3} + ex^{4} + fx^{5} + ---$$

$$y' = b^{0} + 2cx + 3dx^{2} + 4ex^{3} + 5fx^{4} + ---$$

$$\int y\,dx = a^{2}x + \frac{bx^{2}}{2} + \frac{cx^{3}}{3} + \frac{dx^{4}}{4} + \frac{ex^{5}}{5} + ----$$

Since y(0) = 2, therefore, a = 2

and y'(0) = 0, therefore, b = 0

Further, we compare coefficients

x term

$$a + 2*2*2bc = 2$$

$$a = 2$$

x^2 term

$$\frac{b}{2} + 2*2*3bd + 2*4c^{2} = 8$$

$$8c^{2} = 8$$

$$c^{2} = 1$$

$$c = \pm 1$$

x^3 term

$$\frac{c}{3} + 2*2*4eb+2*2*6cd= 7$$

(i) If c = 1

$$24d = 7 - \frac{1}{3} = \frac{20}{3} \quad \text{OR} \quad d = 5/18$$

(ii) If c = -1

$$-\frac{1}{3}(-24d) = 7 \quad \therefore \quad d = -11/36$$

We get multiple solutions. One solution is

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$$y = 2 + x^2 + 5/18x^3 + ----$$

and the second solution is

$$y = 2 - x^2 - 11/36\, x^3 + \text{---------etc.}$$

CONCLUSIONS

i) The Vedic sutras can be very efficiently used for solving even very difficult problems.

ii) The number of steps of operation for multifunction differentiation etc. are drastically cutdown.

iii) The Vedic procedures can be very conveniently applied to very wide range of problems.

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(SSG)

आध्यात्मिक अध्ययन संस्था
(आअस)

★ॐ श्री : ★

ON

VEDIC MATHEMATICS

OR

SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

Maths with Smiles

Basic Concepts

Trigonometry

Coordinate Geometry

वेदात्सर्वं प्रसिद्ध्यति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।

Vedas are the Etermal Source of Knowledge:

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ॐ

प्रातः प्रभृति सायान्तम्
सायन्तः प्रातरन्तम्
यत् करोमि जगन्नाथ
तदस्तु तव पूजनम्

सुबह से शाम तक, शाम से सुबह तक,
मैं जो कुछ भी करता हूँ, हे जगन्नाथ
सब तेरी ही पूजा, तेरी ही उपासना हो जाए।

From morning till eve,
And evening till morn,
Whatever I do ,
Oh, Lord of Universe
Let that be my prayer of thee !

* * * * * * * * * * * * * * * *

ॐ तेजोऽसि तेजोमयि धेहि
वीर्यमसि वीर्यंमयि धेहि
ओजोऽसि ओजोमयि धेहि
बलमसि बलंमयि धेहि
अविराविण्यी ।

हे तेज के पुन्ज हमें तेज दो ।
हे वीर्य के पुन्ज हमें वीर्यवान बनाओ
हे ओजस्व स्वरूप हमें ओजस्व दो
हे बल के पुन्ज हमें बल प्रदान करो
हम अपने स्वरूप की ओर अग्रसर करें ।

★ ★ ★ ★ ★ ★

ॐ सहनाववतु सहनौ भुनक्तु सह वीर्यं करवावहै ।
तेजस्विनावधीतमस्तु मा विद्विषावहै ।
ॐ शान्तिः शान्तिः शान्तिः ।

हे परमात्मा हम गुरू और शिष्य दोनों की रक्षा करो । हम दोनों का पालन करो । हम दोनों साथ में रहकर तेजस्वी दैवी कार्य करें । हम दोनों का किया हुआ अध्ययन तेजस्वी दैवी हो और हम दोनों परस्पर एक दूसरे का द्वेष न करें ।

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2.0 **VINCULUM** (Ref. 2)

2.1 INTRODUCTION:

Vinculum is a unique concept of Vedic Mathematics which is very useful and easy . By using the vinculum approach the computational complexities are drastically cutdown to their bare minimum essential level. At the face of it, it is really very difficult for us to appreciate the real simplification provided by the vinculum approach.

We are generally not familiar with this approach, therefore before using this Vinculum approach for addition, multiplication, division and squaring etc., it is important to become familiar with it. Through this approach we can make it sure that we have almost never to operate digits larger than 5.

2.2 **The Approach**

In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to the next higher order digit.

Illustration 1 : Let us take the number 9. 9 = 10 - 1,therefore 9 is 1 less than 10.This we can also write as $1\bar{1}$, where $\bar{1}$ at the unit place means 1 with a negative sign. For further operations we can use $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply etc. in the usual manner. However, we have to keep in mind that the $\bar{1}$ is always appearing as -1.

In the reverse order, when we need to normalise,

$1\bar{1} = 10 - 1 = 9$

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Illustration 2: 29 = 30 - 1

This can be written as $3\overline{1}$.

Illustration 3: 37 =40 - 3 = $4\overline{3}$

So we find that we take the complement of the digit larger than 5 and write this with a bar indicating the negative sign, and increase the digit at the next higher place.

Illustration 4: 79 = 80 - 1 = $8\overline{1}$

Further 80 = 100 - 20 = $1\overline{20}$

Therefore 79 = $8\overline{1}$ = $1\overline{21}$

So we find 79 can be written as $8\overline{1}$ but since the digit at the tenth place is also very big we can take its complement (from 10, 10 - 8 = 2) and increase the digit previous to this by 1. (0 + 1 = 1, therefore 79 becomes $1\overline{21}$)

It is interesting to note that if there are a few digits larger than 5 appearing together, their collective complement can be directly written by using the Nikhilam Sutra and increasing the previous digit by 1. In this case 9- 7 =2, 10 -9= 1, giving the complement as 21 and the previous digit 0 becomes 0+ 1 = 1. Hence 79 = $1\overline{21}$.

Illustration 5: 3789

By Nikhilam Sutra, the complements of the last three digits are 9 - 7= 2, 9 - 8 = 1, 10 - 9 = 1, and the previous digit gets increased by 1 (3 + 1 = 4). Therefore,

3789 = $4\overline{211}$

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Illustration 6: 286923

The complement of the larger digits (869) by Nikhilam Sutra is 131. Therefore

$286923 = 3\overline{131}23$

Note: that the last digits 2 and 3 at the tenth and the unit place stay on as before, and the digit previous to the bigger ones gets increased by 1.

Illustration 7: 353782

$353782 = 354\overline{22}2$
(The complement of 78 is $\overline{22}$)

Illustration 8: 3728193

$3728193 = 4\overline{3}3\overline{2}2\overline{1}3$

The application of the Vinculum process should be well practised before we use it.However, its use, makes the computation process quite easy and fast.

Illustration 9: 782893

$782893 = 1\overline{22}3\overline{11}3$

Let us further consider a few examples of normalizing the Vinculum number. It shall be surprisingly interesting to learn that the Nikhilam sutra can be used even for the normalizing process.The only difference being that the previous digit is now reduced by 1.

Illustration 10: $3\overline{3}4\overline{4}$

The complement of $\overline{3}$ is 7 and of $\overline{4}$ is 6. Further, each digit before the bar digit gets reduced by 1. So that 3 - 1 = 2 and 4 - 1 = 3.

Therefore $3\overline{3}4\overline{4} = 2736$

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Illustration 11: $3\overline{342}$

Since there are three digits in a group we can use the Nikhilam sutra to straight away write down the complement of this group, and we should remember to reduce the digit previous to this group by 1.

Therefore, by Nikhilam 9- 3 = 6, 9- 4 =5 and 10 - 2 = 8, and 3 - 1 = 2

As such $3\overline{342} = 2658$

Illustation 12: $2\overline{32}6\overline{11}2$

Therefore, $2\overline{32}6\overline{11}2 = 1685892$

Illustration 13: $20\overline{34}0\overline{12}1$

$20\overline{34}0\overline{12}1 = 19659881$

Alternatively, we can normalize the above number in two steps

$20\overline{34}0\overline{12}1 = 2\overline{1}66\overline{1}881$

$= 19659881$

Illustration 14: $3\overline{6}2\overline{27}$

$3\overline{6}2\overline{27} = 24173$

Note: We may not feel very comfortable to see the numbers bigger than 5 appearing with a bar, i.e. in the Vinculum form, as we would not be generally converting the smaller digits into their Vinculum. However, during the computations of the Vinculum numbers we may frequently end up in a situation like this.

Discussion: We shall operate with the Vinculum numbers exactly in the same manner as we do with the ordinary numbers without Vinculum. In other words,we can understand that the property of these numbers is unchanged.However, we handle them alongwith their minus sign in the usual manner.

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Illustration 15: $40\overline{213}1$

Therefore, $40\overline{213}1 = 397871$

In two steps we can say

$$40\overline{213}1 = 4\overline{1}7871$$

$$= 397871$$

Further practise shall give us the grip on the system, which is immensely useful for simplifying the computation processes.

2.3 **Negative Numbers:** A negative number can also be written as a number with a base on all the digits. For convenient operation the bar digits greater than 5 can be converted to +ve digits in the usual manner.

Illustration 16: $-2\ 7\ 4\ 8\ 9 = \overline{2}\ \overline{7}\ \overline{4}\ \overline{8}\ \overline{9}$.

We use Nikhilam sutra to compute the complements of $\overline{7}$ and the pair $\overline{8}\ \overline{9}$. As usual the previous digits have to be decreased by 1, because the bigger bar digits are being converted to the normal +ve form.

$\overline{2}\ \overline{7}\ \overline{4}\ \overline{8}\ \overline{9} = \overline{3}\ 3\ \overline{5}\ 1\ 1$

Illustration 17: $-\ 8\ 0\ 3\ 7\ 9 = \overline{8}\ \overline{0}\ \overline{3}\ \overline{7}\ \overline{9}$.

Further, reducing the bigger digits

$\overline{8}\ \overline{0}\ \overline{3}\ \overline{7}\ \overline{9} = \overline{1}\ 2\ 0\ \overline{4}\ 2\ 1$

Illustration 18: $-\ 7\ 0\ 8\ 2\ 9 = \overline{7}\ \overline{0}\ \overline{8}\ \overline{2}\ \overline{9}$

$$= \overline{1}\ 3\ \overline{1}\ 2\ \overline{3}\ 1$$

Therefore, it is clear that if the first digit of any number is a bar digit, the number will have a negative value. Further, it shall never be possible to convert all digits of such a numbers into

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all positive digits.

2.3.1 The magnitude of such a number can be found directly by making all the digits with a bar. As an alternative we can convert all the digits to the +ve digits and an additional digit $\bar{1}$ shall appear on the L.H.S. and the -ve number can then be obtained by writing the complement of all +ve digits using Nikhilam sutra.

Illustration 19: (i) $\bar{3}\ 7\ 8\ \bar{2}\ 1 = -\ 3\ \bar{7}\ \bar{8}\ 2\ \bar{1}$

$= -\ 2\ 2\ 2\ 1\ 9$

(ii) OR directly we convert the positive digit to bar digits.

$\bar{3}\ 7\ 8\ \bar{2}\ 1 = \bar{2}\ \bar{2}\ \bar{2}\ \bar{1}\ \bar{9}$

as $\bar{3} + 1 = \bar{2}$ and $\bar{2} + 1 = \bar{1}$.

(iii) OR converting bar digits to +ve digits,

$\bar{3}\ 7\ 8\ \bar{2}\ 1 = \bar{1}\ 7\ 7\ 7\ 8\ 1$

Further the complement of all +ve digit shall clearly give the final negative number (-100000 + 77781).

$\bar{1}\ 7\ 7\ 7\ 8\ 1 = 0\ 2\ 2\ 2\ 1\ 9$

Quite clearly the second approach is the easiest and direct.

2.4 **Decimal Numbers :** There is absolutely no new and additional information required for handling the decimal numbers and the decimal point shall continue to remain at the same location.

Illustration 20: $1\ 2\ 0.9\ 8\ 1 = 1\ 2\ 1.\ \bar{1}\ \bar{2}\ 1$

Illustration 21: $0.\bar{3}\ 7\ 2\ \bar{8} = -\ 0.\ 3\ \bar{7}\ \bar{2}\ 8 = -\ 0.2\ 2\ 8\ 8$

OR $0.\bar{3}\ 7\ 2\ \bar{8} = \bar{1}.\ 7\ 7\ 1\ 2 = -\ .2\ 2\ 8\ 8$

OR directly $.\bar{3}\ 7\ 2\ \bar{8} = 0.\ \bar{2}\ \bar{2}\ \bar{8}\ \bar{8} = -\ 0.2\ 2\ 8\ 8$

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2.5 CHECK:

It shall be useful to know that the Vedic check method can be very effectively and efficiently applied to check our conversion to vinculum form, as well as the normalisation. The details are available elsewhere (Ref. 3 & 4). The whole process is just 'Vilokanam' (upsutra No.12), by mere observation although functionally we are utilizing the Sammucchaya sutra (No.15).

3. MULTIPLE CHOICE (Ref. 2)

3.1 NUMBERS :

Vedic Mathematics provides multiple choice for almost everything which we have to do. This makes maths a playful and lively subject which children learn with smiles.Even while handling numbers we have a choice to use the numbers or their complements.

For example $9 = 10- 1=1\overline{1}$

$8 = 10 -2= 1\overline{2}$

$19 = 20 -1= 2\overline{1}$ etc.

Further, if instead of using 19 we use $2\overline{1}$, it is far more easier, simpler, faster and has lesser chances of error.

3.2 ADDITION:

For adding 8 to 9 we can either add in the usual way or we ca add 8 to the complement of 9 ($1\overline{1}$) and get $8 +\overline{1}= 7$ at the unit place and 17 is the result. Similarly for adding 7 to 8, in addition to the normal way we can also add 7 to the complement of 8 ($1\overline{2}$). At unit place $7 + \overline{2} = 5$ and 15 is the result. Small children may find it simpler to reduce 8 by 1 than to add 8 to 9 (Refer Chapter 4).

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3.3 SUBTRACTION:

This can be simply looked as a simple addition of the first number with bar put on all the digits of the number to be subtracted. Again as already illustrated, the process of Shudhikaran can be avoided most of the times by computing the result in the Vinculum form (Refer Chapter 5). Further details are available in Pushp 2.

$$\begin{array}{r} 3\ 2\ 7\ 4 \\ -\ 1\ 4\ 5\ 5 \\ \hline 2\ \bar{2}\ 2\ \bar{1} \\ \hline \end{array} \quad = 1819$$

3.4 MULTIPLICATION:

Even for the simple problem of multiply two numbers, Vedic Maths provides a large number of alternative ways of computing the result. This makes maths a playful subject. Students always like to play games and anybody with whom we play games, we naturally become friendly. And there is no fear or anxiety with a friend.

Example: Multiply 89 x 99

This typical example can be solved in seven different ways.

(i) The easiest solution can be obtained by applying Ekanuyena sutra (Refer Chapter 9).

$$89 \times 99 = 89 - 1 / 99 - 88$$

$$= 88 / 11 = 8811$$

(ii) We can use the Nikhilam method with ease and simplicity (Refer Chapter 7).

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```
89 - 11
99 - 01
--------
88 / 11      =  8811
--------
```

Even here the L.H.S. can be obtained in four different ways.

(iii) We can use the Urdhava sutra and compute the results from right to left (Refer Chapter 10).

```
   89
   99
---------
88   1  1       = 8811
  /16 /8
---------
```

This is the most difficult way and a student of Vedic Maths shall never use this method. But it is interesting to point out that this method is closest to the only method being taught and used by all of us.

(iv) The Urdhava sutra can also be used to compute the result from left to right in two steps.

```
    89
    99
---------
72/  3/ 1     = 8811
 / 15/ 8
---------
```

After a little practice even left to right computations can be done in one line.

(v) It will be much more convenient and easy to first convert the given numbers in their Vinculum form and then apply the Urdhava sutra (refer Chapters 3 & 10). Further details are available in Pushp-2.

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$$\begin{array}{lll} 89 & - & 1\,\bar{1}\,\bar{1} \\ 99 & - & 1\,0\,\bar{1} \\ \hline & & 1/\bar{1}/\bar{2}/1/1 \\ \hline \end{array} \quad = 1\,\bar{1}\,\bar{2}\,1\,1 = 8811$$

It is interesting to observe that there is absolutely no carry over in the entire operation.

(vi) After converting these numbers in the Vinculum form, the multiplication using Urdhava sutra can be done with equal ease and simplicity even from left to right.

$$\begin{array}{c} 1\,\bar{1}\,\bar{1} \\ 1\,0\,\bar{1} \\ \hline 1/\bar{1}/\bar{2}/1/1 \\ \hline \end{array} \quad = \quad 1\,\bar{1}\,\bar{2}\,1\,1 \quad = 8811$$

(vii) The Nikhilam method can also be employed after converting the numbers in their Vinculum form (Refer Chapters 3 & 7).

$$\begin{array}{ll} 1\,\bar{1}\,\bar{1} & +\,\bar{1}\,\bar{1} \\ 1\,0\,\bar{1} & +\,0\,\bar{1} \\ \hline 1\,\bar{1}\,\bar{2} & /\;1\,1 \\ \hline \end{array} \quad = \quad 1\,\bar{1}\,\bar{2}\,1\,1 \quad = 8811$$

NOTE:

There is another unique way of computing from left to right, in which we carrying down as tens the unit digit at each level. This eliminates the requirement of following two steps of working as shown in item (iv).

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3.5 DISCUSSION:

Vedic system provides large number of choices in almost every thing we want to do.The very first step is to recognize the pattern and pick up the most efficient method. This task of pattern recognition alongwith mental working activates the right half of the brain which has the capability of intuition.Thus,with the regular practice of Vedic Maths and its most natural mental procedures, the holistic development of the human brain automatically takes place.Within a couple of years,we shall naturally develop the intuitive faculty -the most powerful facet of the human personality, which unfortunately remains undeveloped in most of the cases.

4. BEEJGANIT

In Vedic Mathematics Beejganit is the original name for algebra. Beej essentially means the seed and Ganit means Mathematics.Therefore, Beejganit should essentially be the seed of different facets of Mathematics.This is quite clear and obvious if one studies mathematics the Ancient Vedic way. Now we shall introduce ourselves to some interesting features.

Like decimal system (the name itself derived from 'Dasham'- meaning ten in Sanskrit) the algebra has also originated from India and in a recent study Swami S.A.Paramhansa has explained how even the name algebra was developed from original Beejganit (presented in the Jaipur Workshop).

Let us consider a polynomial $2x^2 +2x + 4$. If we substitute x = 10 (the deci) we get

$$200 + 30 + 4 = 234$$

Therefore, the coefficients 2, 3 & 4 are nothing but the digits with

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appropriate place value decided by the power of x. So far it is easy to see but Vedic Mathematics carries it ahead in a sweeping manner. Consider the polynomial $2x^3 - 3x^2 -4x + 5$. Apparently, it is difficult to see the clear connection with arithmetic , but after learning the Vedic Vinculum system we can clearly see that this is just another way of writing number $2\ \overline{3}\ \overline{4}\ 5$ (putting x =10). Further, it is not necessary that the coefficients of different terms of x must be all less than the base (here 10).For example

$12x^3 + 11x^2 - 5x + 4$ is equivalent to

$1x^4 + 3x^3 +x^2 - 5x + 4$ which in decimal system is

$12*1000 +11*100 -5*10+4 = 131\overline{5}4$ also

$1*10000 + 3*1000 + 1*100 - 5*10 + 4$

$= 131\overline{5}4 = 13054$

We can further see that x may have any value depending on the unit being used.For example

2 feet 5 inches can also be written as 2x + 5 (here x =12).

2 yard 1 feet can be written as 2x + 1 (here x =3).

This opens up tremendous possibilities for further simplification and ease in different branches of Mathematics. Some of the interesting applications have already been expounded.(Ref. 1, 3, 4, 5,10 etc.). Now we can clearly see, how the Vedic procedures of algebra are directly applicable in arithmetic and vice versa. Division by the Paravartaya sutra (No.4) is one example. Some interesting applications are being presented in other theme papers.

5. DHWAJANK:

The general Vedic method of division based on the Upsutra

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Dhwajank (no.16) reduces the computation process to the bare minimum essential level.Further, the use of Vinculum completely eliminates the process of trial digit. (Ref. 1,3,4,5). A casual reader is very likely to miss the real simplification and ease provided by the Vedic Dhwajank method. Supposing we have two numbers in millions and are trying to compute their ratio as a percentage. In Dhwajank we shall never be dividing by more than 1 or 2 digits. And for a problem like this we shall operate only 3 or 4 digits and all the remaining digits shall remained untouched.

Further, features have been briefly outlined in the other theme papers.

DISCUSSIONS

1. The property of numbers is very extensively exploited in the Vedic Mathematics particularly in the field of computations. The Vedic approach has many special methods - indeed many more of them than conventional methods. This large flexibility of methods finds itself reflected in the mind, when approaching problem from Vedic view point. This indeed is the great benefit of the approach. One wonders at the reason of the supreme simplicity and ease of the Vedic method, which is really lacking in most of our usual methods.

2. Vedic maths is perfectly in oral teaching and mental calculation. They of course demand regular practice of simple problems. They are perfectly in tune with the oral Vedic tradition supplemented by terse statements of important points -called Sutras, which act as an aid to the memory.

3. Even a little exposure to the Vedic Maths approach coupled with some practice clearly shows us that we are dealing with a entirely

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new and direct way of thinking.

4. Vedic Mathematics, due to its multi choice facility, provides wonderful opportunities for the development of the innovative and research faculty of the young students. A large number of Vedic Maths participants have been coming forward with very interesting and useful links. The age and qualification is no barrier in this process. The extensions of the Nikhilam method for multiplying three or more numbers and the use of different bases and subbase have all been provided by one of our UG students.

It shall be interesting to point out the recent research work of Sh. Sanjay, who has just obtained his Engineering Diploma. Mr. Sanjay has very successfully extended, the Vedic method of cubing, to find any power of a given number even beyond the computer capability. Further, he has found that the Urdhava method of solving linear simultaneous equations can be used for solving for even 8 to 10 unknowns using the Pivoting technique. The whole working can be completed in just 2 pages.

5. The Vedic Maths is primarily for mental working and has the potential of developing the human computer to wonderful level where the results of the problems flow very naturally, with the least amount of effort, in the typical Vedic way. This approach provides a corrective methodology to the problem of mental slavery to the calculators.

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TRIGONOMETRY PART -II

1. INTRODUCTION

Vedic Maths has a number of unique features which are absent in the present system. The possibility to carry on the computations from left to right has a great potential for solving a number of those problems which otherwise cannot be directly solved. One such example is the solution of Kepler's equation. The left to right computation approach when combined with the series expansion of sinx, cosx; inverse of tanx; can be conveniently used for computing any of these function for the desired angle or vice versa. Only a few selected examples are presented here. Further detailes are available elsewhere (Ref. 5 & 10).

2. Sine

We know that the series expansion for $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \text{--}$

We may write as $\sin x = P - Q + R - S + \text{-----}$

Here we see that

$Q = Px^2/6,\ R = Qx^2/20,\ S = Rx^2/42$ etc.

which means we can calculate x^2 and then compute each term by multiplying with x^2 and divided by 6, 20, 42 etc.

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Example 1: Compute sin(0.23)

Sin0.23	$(x^2$	.0	5	3	$\bar{1}$		)								0
	x	0 .	2	3											1
	$-x^3/3!$ 6	.	0	0	$\bar{2}$	$-_{2}$	0	$\bar{3}_{1}$	$-_{1}$	$\overset{+}{2}$	$-_{1}$	$\overset{+}{2}_{2}$	$\bar{3}$	$\bar{3}$	2
	$x^5/5!$ 2									5		3	6_{1}	4_{1}	3
	$-x^7/7!$ 42													$\bar{7}_{\overline{14}}$	4
Sin0.23 =		0 .	2	3	$\bar{2}$		0	$\bar{3}$	7		5	3	$\bar{6}$		

For row 2 we multiply rows 0,1 and divide by 6.

$CP^{*}\{{}^{5}_{2}\}=10,\ 10/6 = 2-_{2}$.

$CP\{{}^{53}_{23}\}=21, 21+\overline{20}=1, 1/6 = 0_{1}$.

$CP\{{}^{3\bar{1}}_{23}\}=7, 7+10=17, 17/6 = 3-_{1}$.

$CP\{{}^{\bar{1}}_{3}\}=\bar{3},\ \bar{3}+\overline{10}=\overline{13}, \overline{13}/6=\bar{2}-_{1}$.

For row 3 multiply rows 0, 2 and divide by 2 :

$CP\{{}^{5}_{2}\} = 10,\ 10/2 = 5.$

$CP\{{}^{53}_{20}\} = 6,\ 6/2 = 3$.

$CP\{{}^{53\bar{1}}_{203}\} = 13,\ 13/2 = 6_{1}$.

$CP\{{}^{53\bar{1}}_{03\bar{2}}\}= \bar{1},\ \bar{1} + 10 = 9, 9/2=4_{1}$.

For row 4 : $CP\{{}^{5}_{5}\}$ = 25 and $CP\{{}^{53}_{53}\}$ =30, together these give 280.

$280 / 42 = 7_{\overline{14}}$ $\therefore$ sin0.23 = 0.22797752.

NOTE:*CP denotes cross products.

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3. Cosine

We know that the series expansion for $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + -$

We may write as cosx = 1 +P +Q +R +S +T + ---

where $Q = +P^2/6, R = PQ/15, S = PR/28$ etc. meaning thereby when we compute the value of $x^2/2$ the next term is obtained by squaring this and dividing by 6, the next term is obtained by multipling the two previous terms and then dividing by 15 and so on.

Example 2: Evaluate cos(0.3) radians.

$1 - x^2/2$		$1.0\ \bar{4}\ \bar{5}$	1
$x^4/24$	6	$3_{\bar{2}}\ 3_2\ 7_3\ 5$	2
$- x^6/720$	15	$\bar{1}\ {}_3 0\ {}_3\bar{1}\ {}_2$	3
cos0.3 =		$1.0\ \bar{4}\ \bar{5}\ 3\ 3\ 6\ 5\ \bar{1}$	4
cos0.3 =		0.9 5 5 3 3 6 4 9 to	8 D.P.

Row1 : $\frac{-0.3^2}{2} = 0.0\overline{45}$

Row2: We need to square $\overline{45}$, so we find the duplexes in turn, and divide each of them by 6: $D(\bar{4}) = 16$, $16 / 6 = 3\ r\bar{2}$, put $3_{\bar{2}}$.

$D(\overline{45}) = 40$, 40 + carried $\bar{2}$ (as $\overline{20}$) = 20, 20 / 6 = 3 r2, put 3_2.

$D(\bar{5}) = 25$, 25 + 20 = 45, 45 / 6 = 7_3.

The carried 3 is 30 in the next column, so 30 / 6 = 5.

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Row3: We multiply rows 1,2 : $CP\left\{\begin{matrix}\bar{4}\\ 3\end{matrix}\right\} = \overline{12},\ \overline{12}\ /\ 15 = \bar{1}_3$.

$CP\left\{\begin{matrix}\overline{45}\\ 33\end{matrix}\right\}=\overline{27},\ \overline{27} + 30 = 3, 3\ /\ 15 = 0_3$. $CP\left\{\begin{matrix}\overline{45}\\ 37\end{matrix}\right\}=\overline{43}, \overline{43}+30 = \overline{13},\ \overline{13}\ /\ 15 = \bar{1}_2$.

4. Inverse Tangent :

Similarly, we can use the inverse tangent series for computing the angle whose tangent is given.

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \text{------}$$

Example 3: Compute $\tan^{-1}(0.342)$.

x	0	.	3	4	2				1
$-x^3/3$				$\bar{1}$	$\bar{3}$	$_1$	$\bar{3}$	$_2$	2
$x^5/5$					1	$_{\bar{2}}$	$\bar{1}$	$_1$	3
$\tan^{-1} 0.342 =$	0	.	3	3	0		$\bar{4}$		4
$(x^2$		.	1	$2_{\bar{1}}$	$\bar{3}_6$	$_{\bar{2}}$	)		5
$(x^4$		.	1	4			)		6

We first find x^2 in row 5.

For row 2 we multiply rows 1,5 and divide by 3 :

$CP\left\{\begin{matrix}3\\ 1\end{matrix}\right\} = 3,\ 3\ /\ 3 = 1.$ $CP\left\{\begin{matrix}34\\ 12\end{matrix}\right\} = 10,\ 10\ /\ 3 = 3_1$.

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$$CP\{\begin{smallmatrix}342\\12\bar{3}\end{smallmatrix}\} = 1,1 + 10,11 \,/\, 3 = 3_{2} .$$

For row 3, square row 5 (row 6), multiply by row 1, and divide by 5 :

$$D(1) = 1,\ CP\{\begin{smallmatrix}3\\1\end{smallmatrix}\} = 3,\ 3 \,/\, 5 = 1_{\bar{2}} .$$

$$D(12) = 4,\ CP\{\begin{smallmatrix}34\\14\end{smallmatrix}\} = 16,\ 16 + \overline{20} = \bar{4},\ \bar{4} \,/\, 5 = \bar{1}_{1} .$$

Change the signs of the figures in row 2, and add up the columns.

$$\therefore \tan^{-1} 0.342 = 0.330.$$

5. HYPERBOLIC FUNCTIONS

The hyperbolic functions as well as their inverse functions can also be expressed in the series form and as such they can also be evaluated like the previous example.

Example 4: Evaluate sinh(0.41).

$$\text{We have } \sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \text{----}$$

x		0	.	4	1					
$x^3/3!$	6				1	$_{2}2$	$_{\bar{2}}\bar{5}$	$_{\bar{1}}\bar{1}$	$_{\bar{2}}\bar{3}$	$_{\bar{1}}$
$x^5/5!$	2						1	$_{0}0$	$_{1}\bar{4}$	$_{0}$
$\therefore$ sinh 0.41 =		0	.	4	2	2	$\bar{4}$	$\bar{1}$	$\bar{7}$	

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$\therefore$ sinh 0.41 = 0.42158 to 5 D.P.

Similarly, we can evaluate cosh x and $\tanh^{-1} x$ using:

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots..$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots$$

However, hyperbolic functions are not periodic like the circular functions, so that if x is large a lot of work would be involved. But in such cases we can use the methods to evaluate the following alternative expressions for sinh, cosh and tanh :

$$\sinh x = \frac{1}{2}(e^{-x} - e^{x}); \quad \cosh x = \frac{1}{2}(e^{-x} + e^{x}); \quad \tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

6. **Inverse sine, Inverse cosine and tangent**

Direct series expansion of these functions as well as their hyperbolic functions are not available. But the Vedic Maths provides us with the computation method to compute from left to right. As such we are in a position to compute the most significant digit first which then can be used the next most significant digit and so on. Thus we are in a position to compute the subsequent digits using the digits already obtained. This approach makes it possible to compute inverse of sin using the sin series expansion. Same procedure can also be used even for computing inverse cosine, tangent, inverse sinh, inverse cosh etc.

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Example 5: Evaluate $\sin^{-1}(0.4)$ to 4D.P.

Here we are given sinx = 0.4, and we have to find x.

$$\therefore \quad x = 0.4 + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots.$$

We therefore set up a chart to evaluate the terms on the RHS of this equation :

sinx		0	.	4						row 1
$x^3/3!$	6				1_2	1_4	$6_{\bar{1}}$	1_2		2
$-x^5/5!$	2						$\bar{1}$	$0_{\bar{1}}$		3
x =		0	.	4	1	1	5	1		4
$(x^2$ =			.	$2_{\bar{4}}$	$\bar{3}_{\bar{2}}$	$\bar{1}_{\bar{1}}$	3_2		)	5

The answer appears in row 4, and rows 2,3 and 5 are formed digit by digit as x is found.We therefore work vertically (rather than horizontally as in the last discussion) and as each digit of x is found it is used to obtain next one.

First bring 0.4 down into the answer.Then for row 5, D(4) = 16 = $2\bar{4}$, put $2_{\bar{4}}$. Next, for row 2, multiply rows 4,5 and divide by 6: $CP\{{4 \atop 2}\}$ = 8,8 / 6 = 1_2. Bring down 1 into the answer; then for row 5,

D(41) = 8,8 + $\overline{40}$ = $\overline{32}$, put $\bar{3}_{\bar{2}}$. Row 2 again : $CP\{{4\ 1 \atop 2\ \bar{3}}\}$ =$\overline{10}$, $\overline{10}$ + 20 = 10, 10 / 6 = 1_4.

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Bring down 1 into the answer. Row 5: D(411) = 9,9 + $\overline{20}$ = $\overline{1}_{\overline{1}}$.

Row 2: $CP\left\{\begin{matrix} 4 & 1 & 1 \\ 2 & \overline{3} & \overline{1} \end{matrix}\right\} = \overline{5}$, $\overline{5} + 40 = 35$, $35 / 6 = 6_{\overline{1}}$.

Row 3: We multiply rows 2,5 and divide by 2: $CP\left\{\begin{matrix} 1 \\ 2 \end{matrix}\right\} = 2$, $2 / 2 = 1$, put $\overline{1}$

as this term is negative.

Bring down 5 into the answer.

Row 5: D(4115) = 42, 42 + $\overline{10}$ = 3_{2}.

Row 2: $CP\left\{\begin{matrix} 4115 \\ 2\overline{31}3 \end{matrix}\right\} = 18$, $18 + \overline{10} = 8$, $8 / 6 = 1_{2}$.

Row 3: $CP\left\{\begin{matrix} 1 & 1 \\ 2 & \overline{3} \end{matrix}\right\} = \overline{1}$, $\overline{1} / 2 = 0_{\overline{1}}$.

Bring down 1 into the answer.

$\therefore \quad x = 0.4115$ to 4 D.P.

(the other solution between 0 and 2π is $\pi - 0.4115$)

7. TRANSCENDENTAL EQUATIONS

The method described in the previous section can be applied to the solution of Transcendental equations. The next example shows the solution of Kepler's equation, which is of great importance in positional astronomy. This equation is

$$M = E - e \sin E.$$

where M,E are the mean and eccentric anomalies respectively of a planet in its orbit, and e is the eccentricity of the orbit.

We will take M = 0.30303 radians and e = 0.016722 - which is

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the eccentricity of the Earth's orbit. We want to find E.

$$0.30303 = E - 0.16722 \sin E$$

$$\therefore \quad E = 0.30303 + 0.016722\left(E - \frac{E^3}{3!} + \frac{E^5}{5!} - \ldots\ldots\right)$$

M		0	.	3		0		3		0		3		0		0		1
eE						1	$\bar{4}$	$\bar{5}$	3	2	$\bar{6}$	$\bar{5}$	1	2	1	1	$\bar{3}$	2
$-eE^3/3!$	6									$\bar{1}$	$\bar{1}$	2	$\bar{1}$	$\bar{1}$	3	$\bar{5}$	0	3
$eE^5/5!$	2															4	0	4
E =		0	.	3		1		$\bar{2}$		1		0		1		0		5
(E^2 =			.	1	$\bar{1}$	0	$\bar{4}$	$\bar{5}$	$\bar{1}$	$\bar{1}$	2	3	$\bar{4}$			)		6

where $e = 0.016722 = 0.02\bar{3}\bar{3}22$.

Bring 0.3 into the answer.

Row 2: We multiply e and E : $CP\{{}^{2}_{3}\} = 6 = 1_{\bar{4}}$.

Bring 1 into the answer.

Row 2: $CP\{{}^{2\bar{3}}_{31}\} = \bar{7}$, $\bar{7} + \overline{40} = \bar{5}_{3}$.

Bring down $\bar{2}$ into the answer.

Row 2: $CP\{{}^{2\bar{3}\bar{3}}_{31\bar{2}}\} = \overline{16}$, $\overline{16} + 30 = 2_{\bar{6}}$.

Row 3: We find E^2 (row 6) then multiply rows 2,6, divide by 6 and

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change the sign: $CP\{\begin{smallmatrix}1\\1\end{smallmatrix}\} = 1$, $CP\{\begin{smallmatrix}1\bar{5}\\10\end{smallmatrix}\} = 5$, giving $1\bar{5} = 5$, $5 / 6 = 1\ r\bar{1}$,

put $\bar{1}_{\bar{1}}$.

Bring down 1 into the answer.

Etc. We get E = 0.3081010 to 7D.P.

Although M, and therefore E, take all values between 0 and 2π, by using the angle to the nearest quadrant boundary we can always arrange that the quantity we calculate is less than about $\pi/4$. For example, if M = 3 radians, so that E = 3, we let $E = \pi - A$:

So $\quad M = \pi - A - e\sin(\pi - A) \quad = \pi - A - e\sin A$

from which we can find A, and then E

since $E = \pi - A$.

8. **SOLUTIONS OF TRIANGLES**

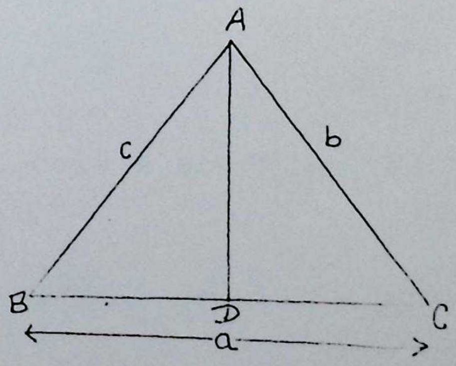
The solution of plane trianlges can be very conveniently done by using the Vedic Tribhujank concept.

Let A, B,C be the three vertices of a triangle and a,b,c be the sides opposite to these vertices. There are only three useful formulae which we use for the solution of plane triangles in addition to the Tribhujank formulae.

1. Kone Nyunena formula (Angle deficiency formula) states that

$$a^2 + b^2 - c^2 = 2a{*}DC$$

where DC is the partial base.



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2. Anurupyena formula : In triangle ABC, if side AB = t and AC=T, and the Tribhujank for angles B and C are (B) q, r, s and (C) Q, R, S then we can easily see that AD = t*r/s and also AD = T*R/S.

Therefore t*r/s = T*R/S this is the Anurupyena formula.

3. Beejank formula: For the triangle ABC,

(B + c)/(b - c) = p(B -C)/p(B + C).

Example 6: If a, b, c are 6,5,4.Find B.

Solution: By Nyunena formula

$$a^2 + c^2 - b^2 = 2a*BD$$

Therefore BD = L.H.S./(2*BC)

= 4*4 + 6*6 - 5*5/2*6

= 9/4

therefore angle B = 9/4, - , 4

= 9, - , 16

If so desired this can further be converted into angle.

Example 7: If a and c are 21 & 10 and angle B is 3,4,5 compute b.

Solution: Since Tribhujank for B is 3,4,5 therefore BD = 10*3/5 = 6 using Nyunena formula

10*10 + 21*21 - b*b = 2*6*21

therefore b = 17.

Example 8: If b = 20 and, angle B and C are 12, 5, 13 and 3,4,5 respectively. Compute C.

Solution: Using the Anurupyena formula C*5/13 = 20*4/5

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Therefore C = 41.6

Example 9: If b = 2 and c = 6 angle C = 3,4,5.Compute angle B.

Solution: Let the Tribhujank of angle B be q,r,s then, using the Anurupyena formula

6*r/s = 2*4/5

Therefore r/s = 4/15

Hence angle B = - , 4, 15

Example 10: If b, c are 2, 3, and angle A is 4,3,5. Find B.

Solution : p(A) = 3 (Beejank)

since (B + C) is supplementary angle of A.

p(B + C) = 1/3

Using Beejank formula

p(B - C)/p(B + C) = (b + c)/(b -c)

p(B - C) = (3 + 2)/[(3 - 2) * 3]

= 5/3

	p(B + C)	1	3	-
	p(B - C)	5	3	-
adding	p(2B)	-4	18	-

Therefore B = -2, 9 (since p(2B) = B)

We can also compute angle C by subtraction.

Example 11: Example 8 can also be done by using the Beejank formulae. Although this approach is unnecessarily long.

p(B) = 5, p(C) = 2

Therefore p(B + C) = 9,7 and

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p(B - C) = 11, -3

Using Beejank formula

(11/-3)*(7/9) = (20 + c)/(20 - c)

77/27 = (c + 20)/c - 20)

$\therefore$ 50c = 2080

c = 41.6

Example 12: In a right angle triangle if a, b are 41, 13 compute angle B.

Solution: Here (B) 41, 13, we know that 41, 13 are the Beejank of a triangle containing an angle 2B.So we can find 2B. 41, 13 is close to 3,1 therefore

	41	13
	3	1
Subtracting	136	-2 = 68, -1

Therefore using table II

2B = .6435 -2/68

= 6141

Therefore B = .3070

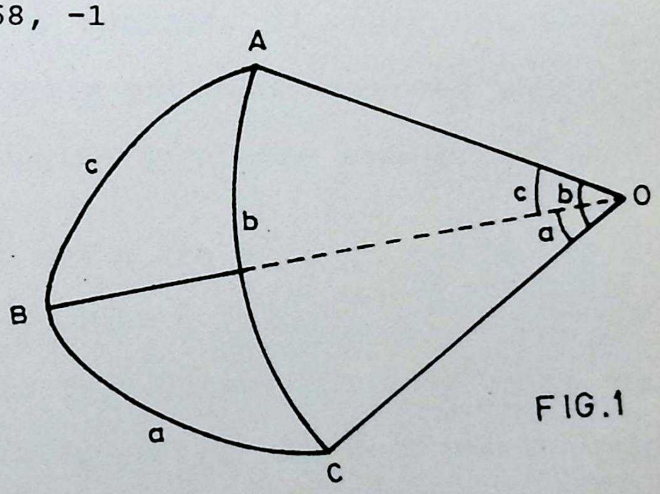

FIG.1

9. **SPHERICAL TRIANGLES**

We know spherical triangle A, B, C is a triangle on the surface of a sphere, the sides being arcs of great circles. The three sides are expressed as angles, they make at the centre of the sphere.

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a = angle BOC, b = angle AOC, c = angle AOB

By angle A we mean the angle between the tangents to the arcs AB and AC at A or the angle between the planes AOB and AOC. (See Fig. 1)

9.1 Tribhujank for Spherical Triangles

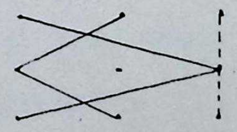
The six angles a, b,c, A,B,C can each be described by a Tribhujank and three of these Tribhujanks are sufficient to define a particular spherical triangle.

a	4	3	5	A	.	.	.
b	5	12	13	B	.	.	.
c	.	.	.	C	8	15	17

Further, we know that two elements of a Tribhujanks are sufficient to define it. We have a set of four formulae which are somewhat complex and are not easy to remember or apply but when we use the Tribhujank approach very simple patterns immerged which can be used to solve the spherical triangles much more easily.

Example 13: If Tribhujanks of a and b are 3, 4, 5 and 5, 12, 13 respectively and angle C is 7, 24, 25. Find c.

Solution: Using cosine formula, this formula is applied when we know two sides and the angle between them. (See Fig. 2) To find the side opposite to the given angle the procedure is given below-



let the Tribhujank of c be x, y, r then by using the above explained

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procedure we have

a	3	4	5
c	7	24	25
b	5	12	13

so, x:r = 3*25*5 + 4*7*12: 5*25*13

= 711 : 1625

Hence (c) 711, -, 1625

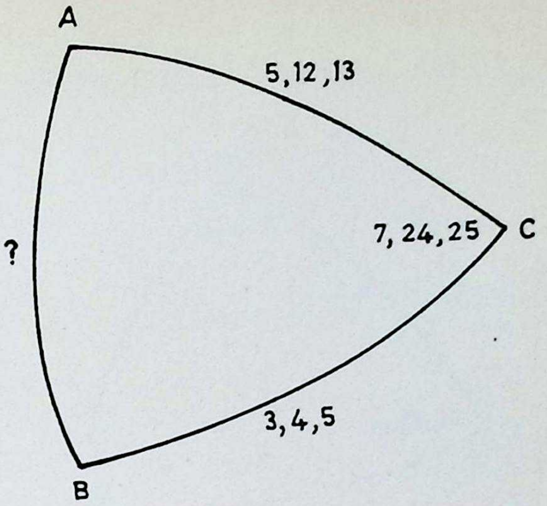


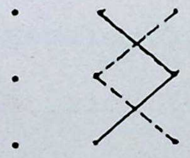
FIG.2

Example 14: Find an angle B between the two sides b & c, whose Tribhujanks are 4,3,5 and 12, 5, 13 and the angle (C) 15, 8,17 opposite to angle C.

Solution : Here

b	4	3	5
c	12	5	13
C	15	8	17

let the Tribhujank of B be x, y, r. By using the sine formulae



we get y:r = 13*3*8: 5*5*17

∴ (B) -, 312, 425

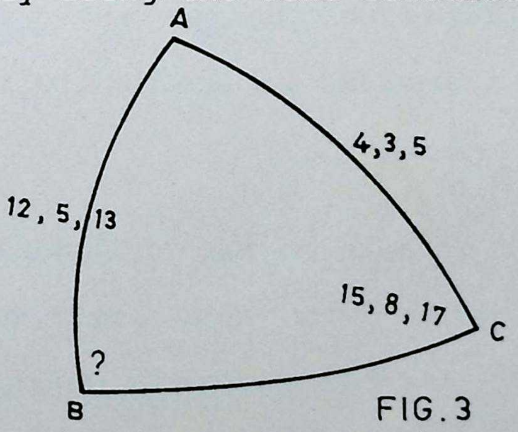


FIG.3

Example 15: Find the angle B by using the Polar Cosine rule if Tribhujanks of b and c are 12, 5, 13 and 8, 15, 17 and the angle (A) 3, 4, 5.

Solution: Polar cosine rule states that if we have two angles and

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the common side and require the angle opposite the side, or if we have three angles and requires a side.

The formula is

Given

A	3	4	5
b	12	5	13
c	8	15	17

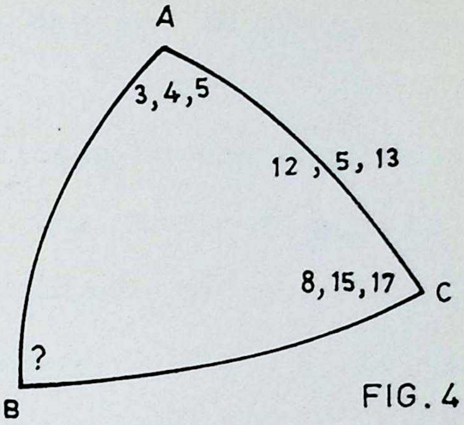
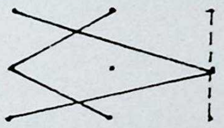


FIG.4

hence angle B is x/r = (12*4*15- 13*3*8)/5*13*17

= 408/1105



∴ angle (B) 408, -, 1105

10. Spherical Triangles using Beejanks: In the previous section we have seen how conveniently we can find out the Tribhujank of the remaining angles if we are given the Tribhujank of any three angles. However,if we are given the angles in radians or degrees it shall be more convenient to use the Beejanks.The given angles can be converted to their Beejank by using the methods outlined in the previous section.The pattern of multiplications is extremely simple, easy and straightforward.

11. Planetary Positions :Further, Tribhujank can be used to find the position of a planet at the desired time, when certain basic data has been given. There can be three types of problems namely-

i) Heliocentric position - Finding the position of a planet in its orbit around the Sun.

ii) Geometric Position - Finding the position as seen from the earth.

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iii) The Planet Finder - Locating the direction of the Planets, Sun, Moon and Galaxy at any time of the day and night and for any observer on the earth.

All these problems alongwith the Eclipses of the Sun, Moon etc. have been very precisely computed by the Ancient people. The Tribhujanks can be very effectively utilised to carry on these computations without the use of computing device (Ref. 6, 14, 15).

CONCLUSIONS:

i) The Vedic sutras provide very simple and easy methods for handling even complicated problems.

ii) The number of formulae required is drastically cut down.

iii) The Tribhujanks and Beejanks can be systematically applied to very wide range of problems.

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COORDINATE GEOMETRY

1. EQUATION OF THE LINE

The equation of a line passing through two points can be written directly by using sutra Paravartaya Yojyet (No. 4)by a mere casual look at the coordinates. The constant term can also be obtained by using the upsutra Adyam Antyam and Madhyam (No. 3) which means the product of the means minus (-) the product of the extremes (Ref. 1).

Example 1: Find the equation of a line passing through two points (8,9) and (4,5).

$$\begin{array}{cc} 8 & 9 \\ 4 & 5 \\ \hline \end{array}$$

$$(8-4)y = (9-5)x + (8*5-9*4)$$

$$\therefore 4y = 4x + 4$$

$$y = x + 1$$

The coefficients of x and y are the difference of the y & x coordinates respectively and the constant term is the determinant of the coordinates.

Example 2: Find the equation of a line passing through (3,5) and (2,3)

$$\begin{array}{cc} 3 & 5 \\ 2 & 3 \\ \hline \end{array}$$

$$(3-2)y = (5-3)x + (3*3-2x5)$$

$$\therefore y = 2x - 1$$

Example 3: Find the equations of the lines through (2,3) which make an angle of 45° to the line 2y = x +1. There are two lines the

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Tribhujank for the given line and 45° are

line	2	1	-
45°	1	1	- by adding
	1	3	-

Therefore $y = 3x + C$

By subtracting

	3	-1	-

Therefore $3y = -x + C$

Since the lines pass through the point (2,3) we get $y = 3x - 3$ and $3y = -x + 11$.

Example 4: Find the equation of the line through the point (2,3) which makes an angle of 45° to the line $y = x + 1$. There are two lines: the Tribhujanks for the given line and 45° are

line	1	1
45°	1	1
By adding	0	2

Therefore the line is $0*y = 2x + C$ $\therefore 2x + C = 0$

By subtracting: 2 0 $\therefore$ line $2y = C$

Since the lines pass through the point (2,3) we get $2x - 4 = 0$ and $2y = 6$.

We can see that how convenient it is to solve this class of problems by using the Tribhujanks.

2. **EQUATION OF CIRCLE**

Example 5: Find the equation of circle with a centre (2,3) which is tangential to the line $y = 3x$.

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We need the radius of the circle, AC, since the circle is tangential to the line y = 3x, the sum of the angles between the centre and the x-axis (E) and the centre and the tangent (F) has a gradient of 3. Therefore the second angle (E) and hence the radius of the circle can be obtained by finding the difference of the Tribhujanks (E + F) and E. (See Fig. 1).

Tangential line (E + F)	1	3	$\sqrt{10}$
Centre (E)	2	3	$\sqrt{13}$
(E + F) -E	11	3	$\sqrt{130}$

OR $11/\sqrt{10}$, $3/\sqrt{10}$, $\sqrt{13}$

Since OC $=\sqrt{13}$ $\therefore$ AC $= 3/\sqrt{10}$

Therefore the equation is

$$(x-2)^2 + (y-3)^2 = 9/10$$

Since we are interested in the actual lengths therefore the Tribhujank has to be divided by suitable number (here $\sqrt{10}$).Further, frequently we do not require all the three elements of the Tribhujank.

3. LENGTH OF PERPENDICULAR

Example 6: What is the length of perpendicular from the point (4,3) to the line y = 2x.

Solution: If we subtract the Tribhujank for point from the Tribhujank for the line we shall get the Tribhujank for the remaining angle.The middle element of which shall be the desired length of perpendicular (See Fig. 2).

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EX. 5

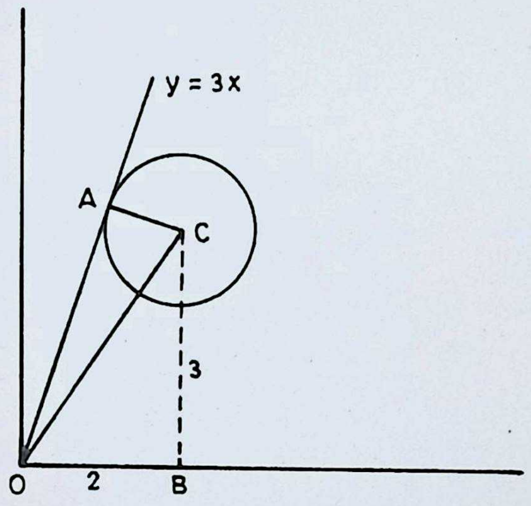

FIG. 1

EX. 6

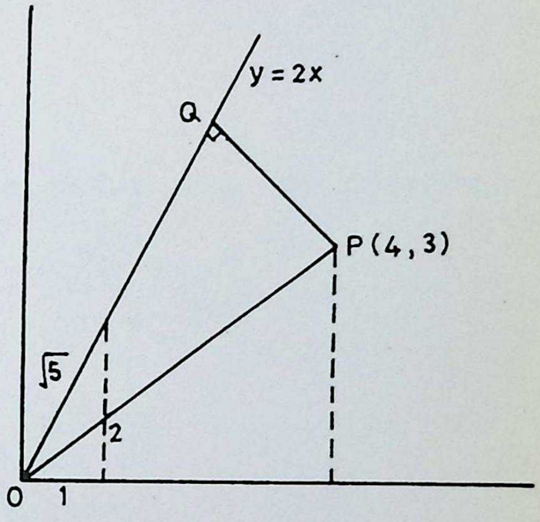

FIG. 2

EX. 9

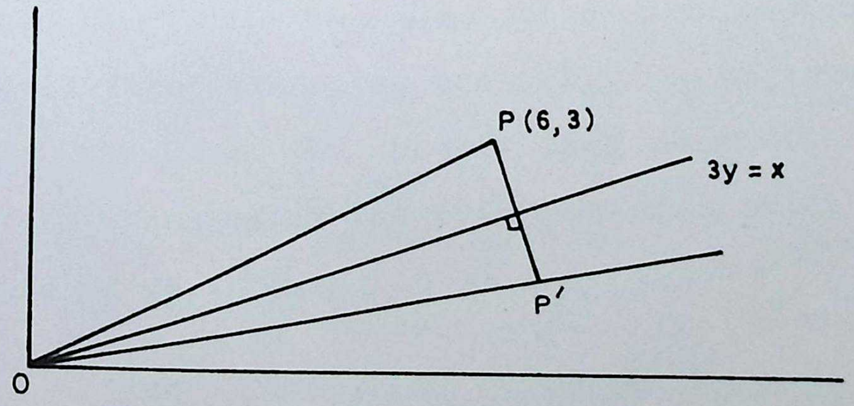

FIG. 3

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line	1	2	$\sqrt{5}$
point	4	3	5
Subtracting:	10	5	$5\sqrt{5}$

Triangle OPQ $=10/\sqrt{5}$ $\quad 5/\sqrt{5}$ $\quad 5$

Therefore PQ = $\sqrt{5}$

Example 7: Compute the distance of the point (-2,-3) from the line 4y + 3x = 0.

line	4	-3	5
point	-2	-3	-
Subtracting	-	18	-

Therefore PQ = 18/5 = 3.6

Example 8: Find the distance of p (5,4) from the line 3y + 4x = 7.

Solution : This line does not pass through the origin, therefore we should first transpose the origin to any convenient point which lies on the line. We observe that point (1,1) lies on the line therefore transposing the origin to this point we have to find the distance of (4,3) from 3y + 4x = 0.

line	3	-4	5
point	4	3	-
Subtracting	-	-25	-

Therefore PQ = -25/5 = -5 = 5 (we ignore the sign).

NOTE: We know that if we transpose the origin to any point then

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the coordinates of all other points are obtained by subtracting the coordinates of the new origin. No adjustment is required due to the transposition of the origin.

4. FOOT OF PERPENDICULAR

Example 9: Find the coordinates of the foot of the perpendicular from P(6,3) onto the line 3y = x.

Solution : We compute the Tribhujank for the reflection P′ of the point P in the given line and then the mid point of PP′ shall be the desired foot of the perpendicular. (see Fig. 3).

line	3	1	$\sqrt{10}$
2*line	8	6	10
p	6	3	-
Subtracting we get p	66	12	-

Therefore p′ 66/10 12/10

Therefore the foot of the perpendicular shall be

$$\left(\frac{6 + 6.6}{2}, \frac{3 + 1.2}{2}\right) = (6.3,\ 2.1)$$

5. ANGLE BETWEEN TWO LINES

The angle between any two lines can be obtained by simple subtraction of their Tribhujanks.Since we are interested only in the slope of the line therefore the absolute terms can be disregarded.Further, the order of subtraction is not important.

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Example 10: Find the angle between the lines $3y + 4x = 6$ and $y + x = 4$.

Solution : Tribhujank

line	3	4	5
line 2	1	1	$\sqrt{2}$
Subtracting	7	1	$5\sqrt{2}$

The Beejank of the desired angle is 7 or p = 7.

Example 11: Find the angle between $y = 3x + 2$ and $3y = -x -3$.

line 1	1	3	$\sqrt{10}$
line 2	3	- 1	$\sqrt{10}$
Subtracting	0	10	10
or	0	1	1

Therefore these lines are perpendicular to each other.Since 0,1 is the Beejank for 90°.

6. TRANSFORMATIONS IN A PLANE

The Vedic Tribhujank approach can be very efficiencfly utilised for rotations and reflections in a plane as well as in space. However, since we are now concerned with the actual lengths, we must carefully multiply or divide our Tribhujank as necessary.

6.1 Rotations:

Rotations of a point, line or curve through a given angle can be very conveniently carried out by using the Vedic Tribhujank approach.

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Example 12: Rotate P(4,3) through an angle of 90° anticlockwise by using origin as the centre of rotation.

Tribhujank for P(4,3) and 90°.

P	4	3	5
90°	0	1	1
Adding	-3	4	5

Therefore the rotated point is P′ (-3,4).

Example 13: Rotate P(-3,5) 90° clockwise about the point (2,7).

We transpose the origin to the point of rotation (2,7). The new coordinates of P are (-3 -2, 5-7) =(-5,-2). Now we rotate clockwise this point about(0,0) by 90°.

p	-5	-2	-
90°	0	1	-
Subtracting	-2	5	-

Therefore the rotated point p′(-2,5) relative to original origin is P′(-2+2,5+7) = (0,12).

Example 14: Rotate the point P(4,3) through an angle (A) 3,4,5 about origin in the anticlockwise direction.

Solution:

P	4	3	5
A	3	4	5
Adding	0	25	25

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But since we want to keep OP′ = OP i.e. the distance of the points of the origin is unchanged, as such we divide by 5 and the new point is P′(0,5).This lies on the y axis we should be expecting this result as we have rotated the point through the complement of the angle it is making with the x axis. The new angle has Beejank p,q = 0,1.Further, even the rotation by an obtuse angle would be done in exactly the same manner.

NOTE:

(i) It shall be useful to learn that for plotting an equiangular spiral starting with a point (1,0) we just need to rotate it through an angle of 45° by adding the Tribhujank 1,1, $\sqrt{2}$. The first two elements when plotted and joined would give the desired spiral.There is no need to normalize the distances.

(ii) The successive integration or differentiation of the function cosx (represented by the x-axis) is simply the rotation of the OX axis in the anticlockwise or clockwise direction through 90° each time.

$$\frac{d}{dx}(\cos x) = -\sin x \text{ (clockwise)}$$

$$\int \cos x\,dx = \sin x + c \text{ (anticlockwise)}$$

Example 15: Rotate the line y = 5x + 2 through the angle 3,4,5 in the anticlockwise direction, about the origin.

Solution: Let P(x,y) and P′(x,y) represent the same point P on the line before and after rotation respectively. Since we are rotating about the origin OP shall be equal to OP′ (say = z).The

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Tribhujanks for the two lines shall be x, y,z and X,Y,z, respectively. If we subtract Tribhujank 3,4,5 from X,Y,z, we should get x,y,z.

	X	Y	z
	3	4	5
Subtracting	3X+4Y	3Y-4X	5z
Or	(3X+4Y)/5 ,	(3Y-4X)/5,	z
Also subtracting	x	y	z

Therefore $x = (3X + 4Y)/5$

$y = (3Y - 4X)/5$

Therefore the equation for the transformed line shall be $y = 5x + 2$. Substituting

$$(3Y - 4X)/5 = 5(3X+4Y)/5 + 2$$

OR $3Y - 4X = 5(3X+4Y) + 10$

$\therefore 19X + 10 + 17Y = 0$

OR which we may write as

$$19x + 17y + 10 = 0$$

Example 16: The previous example can also be done directly in a single step by rotating the given line through the desired angle. For anticlockwise rotation this is accomplished by simply adding the Tribhujank of the angle of rotation to the equation of the line using the Urdhava sutra (No. 3) "Vertically and Crosswise".

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y = 5x + 2

3 4 5

Adding(1*3y-5*4) = (3*5+1*4)x + 10

$- 17y = 19x + 10$ is the desired equation.

Example 17: Rotate the curve $y = x^2$ through an angle 3,4,5.

Solution: The substitution for a rotation of 3,4,5 was found in the Example 15.Substituting the same.

$$(3y - 4x)/5 = (3x + 4y)^2/5*5$$

OR $15y - 20x = 9x^2 + 16y^2 + 24xy$

or $9x^2 + 16y^2 + 24xy - 15y + 20x = 0$ is the equation of the rotated curve.

NOTE: Similarly we can see that if we rotate this curve in the clockwise direction by 90° the equation of the rotated curve shall be $x = y^2$.

6.2 **Reflections:**

We may be interested to reflect a point in the coordinate axis, a line parallel to the axis, or an inclined line which may or may not be passing through the origin.

(i) When we reflect a point in one of the axis, the same coordinate remains unchanged and the other coordinate changes sign by using the Paravataya sutra (No.4).

Example 18: Reflect the point P(-4,5) in y axis.

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Solution : The y coordinate is unaltered therefore P′ (4,5).

(ii) If a point is reflected in a line parallel to the axis; we transpose the origin and reflect.Then transpose back.

Example 19: Reflect (-4,3) in the line y = 5.

Solution: We transpose the origin and reflect P(-4,-2), in y = 0 in the new x-axis to get P′(-4,+2). Then add y = 5, to get P′(-4,7).

```
P'*    |
-----------------------------------
P *    |
---------------------------------
       |
       |
```

(iii) If we reflect a point in an inclined line we subtract the point Tribhujank from the line Tribhujank and then add the line Tribhujank to the remainder.Since the distance of the point from the origin remains constant, we have to divide by the square of the third element of line Tribhujank.

Example 20: Reflect P(3,3) in the line y = 3x.

Solution: The line Tribhujank is 1,3, $\sqrt{10}$.

line	1	3	$\sqrt{10}$
P	3	3	$\sqrt{8}$
Subtracting	12	6	$3\sqrt{20}$
line	1	3	$\sqrt{10}$
Adding	-6	42	$30\sqrt{2}$

Further,dividing by the square of the third element of the line Tribhujank. Therefore

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P′ (-6/10, 42/10) or P′(-0.6, 4.2)

The third column need not be completed.

Example 21: As usual with the vedic maths we can solve this problem in an alternative way.

If we find the double of the line Tribhujank and subtract the point Tribhujank from it, we shall get the desired Tribhujank and hence the reflection.

line	1	3	$\sqrt{10}$
2*line	-8	6	10
P	3	3	-
Subtracting	-6	42	-

Therefore P′(-6/10, 42/10)

OR (-0.6, 4.2)

(iv) If we reflect a point in the line of the form y = ax+C; not passing through the origin, we first transpose the origin to point where the line intersects the y-axis (0,C) and reflect the transposed point as usual.

Example 22: Reflect P(3,3) in the line y = 3x + 2.

Solution: This line intersects the y-axis at the point (0,2). As a first step we transpose the origin to this point and consider the reflection of (3,1) in the line y = 3x.

line	1	3	$\sqrt{10}$
2*line	-8	6	10
P	3	1	-
Subtracting	-18	26	-

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Therefore (-18/10, 26/10) or (-1.8, 2.6)

To which we add back p'(-1.8, 4.6). (0.2) to get

Discussion:

1. For reflection in a generasl line ay = bx + C,we transpose the line to a convenient point (x1, y1) on the line (where x1 & y1 are integer numbers).So that the line becomes ay = bx and rest of the procedure is the same.

2. Further, for enlargement of a geometrical figure by a factor a, all the coordinates are multiplied by the factor a.

Example 23: Reflect P(3,3) in the line 2y = 3x + 4.

More origin to a suitable point (2,5) on the line and reflect P(1, -2) in 2y = 3x.

line:	2	3	$\sqrt{12}$
2*line:	-5	12	13
Point:	1	-2	-
Subtracting	-29	2	-

OR (-29/13, 2/13) is the reflected point.

Therefore P' (-3/13, 67/13)

Even if we shift the origin to (0,2) which lies on this line, the same reflected point is obtained.

7. COORDINATE GEOMETRY IN 3D

The usual form for the equations of a line and a plane are

line: $$\frac{x - p}{a} = \frac{y - q}{b} = \frac{z - r}{c},$$

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where (p,q,r) is a point on the line and P(a,b,c) is the point such that OP is in the direction of the line.

plane: $ax + by + cz + d = 0$,

where P(a,b,c) is the point such that OP is perpendicular to the plane.

In general, a point in space can be described by four numbers such that the first three are x,y,z the coordinates of the point and the fourth r is the length of the line joining this point with the origin. Thus the point P (2,2,1) can be described by the quadruple A(2,2,1,3) as 3 is the distance from the origin, i.e.,

$$OP = \sqrt{2*2 + 2*2 + 1*1} = 3.$$

We can extend the Tribhujank formulae to compute the Tribhujank for an angle in space. If

A	x	y	z	r
B	x	y	z	R
Angle AOB by subtraction	$(xX+yY+zZ)$	$+\sqrt{(xY-yX)^2+(xZ-zX)^2+(yZ-zY)^2}$		rR

The Urdhava sutra (No 3) is at work.

The middle element can also be obtained from the other two by the use of the Tribhujank theorem.We use the Sankalan sutra (No.7) for computing the middle element.

Example 23: Find the Tribhujank for angle AOB where

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A	2	2	1	3
B	2	3	6	7
Subtracting Angle AOB	(2*2+2*3+1*6)	$\sqrt{(2*3-2*2)^2 + (2*6-1*2)^2 + (2*6-1*3)^2}$		3*7
	16	$\sqrt{185}$		21

Example 24: Find the angle between the planes 2x + y - 2z = 3 and 3x - 6y - 2z = 4. The angle between the planes is the same as the angle between their normals.

Therefore, we need POQ where P is the point (2, 1, -2) and Q is the point (3,-6, -2):

P	2	1	-2	3
Q	3	-6	--2	7
∠POQ	4	-		21

which is the required Tribhujank.

Example 25: Find the angle between the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}$.

We can transpose the origin to the point of intersection of the lines and find the angle between $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$

1	2	3	$\sqrt{14}$
2	3	4	$\sqrt{29}$
20	-	-	$\sqrt{14*29}$

is the required Tribhujank.

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Example 26: Find the perpendicular distance of the point (2,1,-3) from the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-1}$.

We know (1,2,0) lies on the line, and we transpose the origin to that point and then find the distance of P(1, -1, -3) from

$\frac{x}{2} = \frac{y}{-1} = \frac{z}{-1}$.

1	-1	-3	$\sqrt{11}$
2	-1	-1	$\sqrt{6}$
6	$\sqrt{30}$		$\sqrt{66}$

Since OP = $\sqrt{11}$

We divide through by $\sqrt{6}$

$\therefore$ PQ = $\sqrt{5}$

Example 27: If A(1,7,3), B(2,1, -3) show that OA, OB are perpendicular.

The Tribhujank for 90° is 0,1,1 so in finding angle AOB we require that the vertical products forming the first number of the Tribhujank should add up to zero.

1	7	3	-
2	1	-3	-
2+7-9	-		-

which they do.

Hence angle AOB = 90°

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8. TRANSFORMATIONS IN 3 DIMENSIONS

8.1 Translation

Example 28: Find the coordinates of the point (6, 1, 5) if the origin is transposed to the point (1, 2, 3).

It is often useful to translate the origin of coordinates to some other point, and as explained for the 2-dimensional case , we subtract the coordinates of the point which is to become the new origin from those of the other point or points.

∴ relative to the new origin (6, 1, 5) becomes (5, -1, 2).

8.2 Rotation about one of the Coordinate Axes

If we are rotating a point about, say, the x-axis, the x-coordinate of the point will not change and the rotation is in a plane parallel to the yz plane. This is therefore really a problem in 2-dimensions, and we can use the method described earlier.

Example 29: Rotate P(4,2,3) by angle 4,3,5 about(a)the y-axis (b) the z-axis.

(a) The y-coordinate of P is unchanged.

```
4    2    3    -
4    -    3    5  by adding
----------------------
7         24
-    2    --
5          5                  ∴   P'(1.4, 2, 4.8)
----------------------
```

(b) the z-coordinate is unchanged.

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```
4    2    3    -

4    3    -    5   by adding
--------------------
10   20
--   --   3                    .
 5    5                       . .  P'(2, 4, 3)
--------------------
```

8.3 Rotation of 90°

Example 30: Rotate the point (9, 4, -5) through an angle of 90° in the plane whose normal is given by the quadruple 2,3,6,7.

Since the line OP lies in the plane, and the normal is perpendicular to it, the normal is perpendicular to OP. This is confirmed by the total of the three vertical products below -

```
2*9 + 3*4 + 6*(-5) = 0

     2           3           6

     9           4          -5
  ---------------------------
  3*-5-6*4   6*9-2*-5   2*4-3*9    = -39, 64, -19
  ---------------------------
```

$\therefore$ the rotated point is P' $\frac{1}{7}$(-39, 64, -19).

that is, for the x-coordinate we take the cross-product in the 2nd and 3rd columns, for y we take the cross-product (in the reverse direction) in the 1st and 3rd columns, and for z the cross-product in the 1st and 2nd columns. All of these must be divided by 7, the fourth element of the quadruple.

In general, for 90° rotations, if the point is p, q, r and the quadruple defining the normal direction is a,b,c,R, then the rotated point has coordinates-

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```
    a          b          c

    p          q          r
------------------------------
 1
 -(br - cq, cp - ar,  aq - bp)
 R
------------------------------
```

8.4 Rotation in General

Example 31: Rotate the point P(3, 4, -3) through the angle 4, 3, 5 in the plane whose normal is given by 2,3,6,7.

```
     Given a normal a    b    c    R

           a point  p    q    r

     and a rotation      l    m    n
```

```
                      1
then P' is given by  - [m(br-cq)+plR, m(cp-ar)+ qlR, m(aq -bp)+rlR]
                      nR
```

Here we have

```
                      2              3          6        7

                      3              4         -3

                                     4          3        5
                  --------------------------------------------
                  1
                  --(3*(-33)+ 3*28, 3*24+4*28, 3*(-1)-3*28)
                  35
                  --------------------------------------------
```

$$\therefore P' = \frac{1}{35}(-15,\ 184,\ -87)$$

CONCLUSIONS

i) The problems of coordinate geometry (both 2-D and 3-D) can be handled much more easily and conveniently by using the Vedic Tribhujank system.

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ii) The long list of formulae which we use at present is drastically cut down.

iii) The simplification provided in the transformations seems to be having good potential for applications in the Computer Graphics.

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